# 28-th Mathematical Olympiad 1992

*January 1992*

Time allowed: 3.5 hours

1. (a) Observe that the square of 20 has the same number of non-zero digits as the original number. Does there exist a two-digit number *other than* 10, 20, and 30 the square of which has the same number of non-zero digits as the original number? If you think there is one, then give an example. If you claim that there is none, then you must prove your claim.

   (b) Does there exist a three-digit number other than 100, 200, and 300 the square of which has the same number of non-zero digits as the original number?

2. Let $ABCDE$ be a pentagon inscribed in a circle. Suppose that $AC$, $BD$, $CE$, $DA$, and $EB$ are parallel to $DE$, $EA$, $AB$, $BC$, and $CD$ respectively. Does it follow that the pentagon has to be regular? Justify your claim.

3. Find four distinct positive integers the product of which is divisible by the sum of every pair of them. Can you find a set of five or more numbers with the same property?

4. Determine the smallest value of $x^2 + 5y^2 + 8z^2$, where $x$, $y$, and $z$ are real numbers subject to the condition $yz + zx + xy = -1$. Does $x^2 + 5y^2 + 8z^2$ have a *greatest* value subject to the same condition? Justify your claim.

5. Let $f$ be a function mapping positive integers into positive integers. Suppose that
$$f(n+1) > f(n) \qquad \text{and} \qquad f(f(n)) = 3n$$
for all positive integers $n$. Determine $f(1992)$.