16-th British Mathematical Olympiad 1980

March 13, 1980

Time: 3.5 hours.

1. Prove that the equation $x^n + y^n = z^n$, where $n > 1$ is an integer, has no solution in integers x, y, z with $0 < x, y \leq n$.

2. Find a set $S = \{n, n+1, \dots, n+6\}$ of seven consecutive integers for which there is a polynomial $P(x)$ of degree 5 with integer coefficients such that $P(n) = n$ for five elements of S, including the least and greatest, and $P(n) = 0$ for one element of S.

3. On the diameter AB on a semicircle there are two points P and Q, and on the semicircle there are two points R and S such that $PQRS$ is a square. Let C be a point on the semicircle such that the areas of triangle ABC and square $PQRS$ are equal. Prove that a line containing one of the points R, S and one of the points A, B intersects a side of the square at the incenter of the triangle.

4. Find all real numbers a_0 for which the sequence $(a_n)_{n=0}^{\infty}$ given by $a_{n+1} = 2^n - 3a_n$ for $n \geq 0$ is strictly increasing.

5. In a party of ten persons, among any three persons there are two who do not know each other. Prove that at the party there are four persons, no two of which know each other.

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