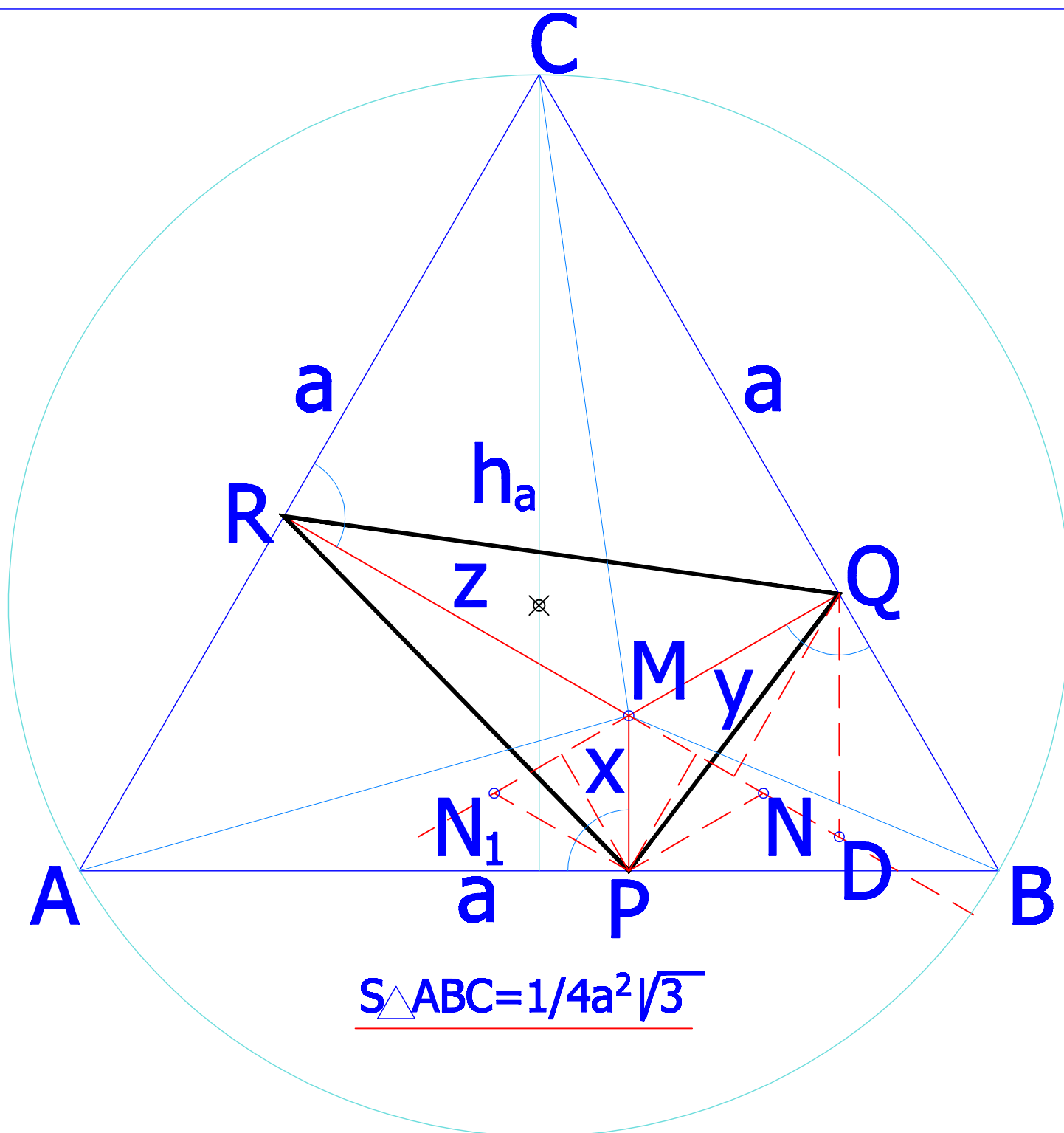




Дадено:

Равностранен триъгълник ABC

$$S_{\triangle ABC} = 27\sqrt{3}$$

$$x:y:z = 1:2:3$$

$$S_{\triangle PQR} = ?$$

Решение: /Геометричен начин/

$$S_{\triangle ABC} = 27\sqrt{3} = a \cdot h_a / 2 \quad a \cdot h_a / 2 = 27\sqrt{3}$$

$$a = 2 \cdot 27\sqrt{3} / h_a$$

$$h_a = 2 \cdot 27\sqrt{3} / a$$

$$S_{\triangle ABC} = 1/4 a^2 \sqrt{3} = 27\sqrt{3}$$

$$1/4 a^2 = 27$$

$$a = \sqrt{27 \cdot 4}$$

$$h_a = 2 \cdot 27\sqrt{3} / \sqrt{27 \cdot 4}$$

$$h_a = 27\sqrt{3} / \sqrt{27}$$

$$h_a = \sqrt{3 \cdot 27} = 9 \quad a = 2 \cdot 27\sqrt{3} / 9 = 6\sqrt{3}$$

$$S_{\triangle ABM} = a \cdot x / 2$$

$$S_{\triangle BMC} = a \cdot y / 2$$

$$S_{\triangle AMC} = a \cdot z / 2$$

$$a \cdot x / 2 + a \cdot y / 2 + a \cdot z / 2 = a \cdot h_a / 2$$

$$x + y + z = h_a$$

$$x + y + z = 9$$

$$y:z = 2:3 \quad x:z = 1:3$$

$$y = 2/3 \cdot z \quad x = 1/3 \cdot z$$

$$1/3 \cdot z + 2/3 \cdot z + z = 9$$

$$z \cdot (1/3 + 2/3 + 1) = 9$$

$$z = 4,5 ; y = 3 ; x = 1,5$$

$$MN = x = 1,5$$

$$S_{\triangle PMN} = x \cdot h_x / 2 = 1/4 \cdot x^2 \cdot \sqrt{3}$$

$$h_x = 1/2 \cdot x \cdot \sqrt{3} = 1/2 \cdot 1,5 \cdot \sqrt{3} = 0,75\sqrt{3}$$

$$S_{\triangle PMR} = z \cdot h_x / 2 = 4,5 \cdot 0,75 \cdot \sqrt{3} / 2$$

$$S_{\triangle PMR} = 1,6875 \cdot \sqrt{3}$$

$$S_{\triangle DMQ} = y \cdot h_y / 2 = 1/4 \cdot y^2 \cdot \sqrt{3}$$

$$h_y = 1/2 \cdot y \cdot \sqrt{3} = 1,5 \cdot \sqrt{3}$$

$$S_{\triangle QMR} = z \cdot h_y / 2 = 3,375 \cdot \sqrt{3}$$

$$S_{\triangle PMQ} = y \cdot h_x / 2 = 1,125 \cdot \sqrt{3}$$

$$S_{\triangle PQR} = S_{\triangle PMR} + S_{\triangle QMR} + S_{\triangle PMQ}$$

$$S_{\triangle PQR} = 1,687 \cdot \sqrt{3} + 3,375 \cdot \sqrt{3} + 1,125 \cdot \sqrt{3} = 6,187 \cdot \sqrt{3} = 10,717$$