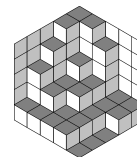




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- [1] Let H be the orthocenter of an acute-angled triangle ABC . The circle Γ_A centered at the midpoint of BC and passing through H intersects the sideline BC at points A_1 and A_2 . Similarly, define the points B_1, B_2, C_1 and C_2 .
Prove that six points A_1, A_2, B_1, B_2, C_1 and C_2 are concyclic.
- [2] (i) If x, y and z are three real numbers, all different from 1, such that $xyz = 1$, then prove that
$$\frac{x^2}{(x-1)^2} + \frac{y^2}{(y-1)^2} + \frac{z^2}{(z-1)^2} \geq 1.$$
 (With the $\sum$ sign for cyclic summation, this inequality could be rewritten as $\sum \frac{x^2}{(x-1)^2} \geq 1$.)
(ii) Prove that equality is achieved for infinitely many triples of rational numbers x, y and z .
- [3] Prove that there are infinitely many positive integers n such that $n^2 + 1$ has a prime divisor greater than $2n + \sqrt{2n}$.