LAST THEOREM OF P. FERMAT

(with other topics)

(1) Algebraic Proof of Last Theorem of P. Fermat (1601-1665AD)--Part I.

(2) Geometrical Proof of Last Theorem of P. Fermat (1601-1665AD)--Part II

(3) Compressed Circle--Part III

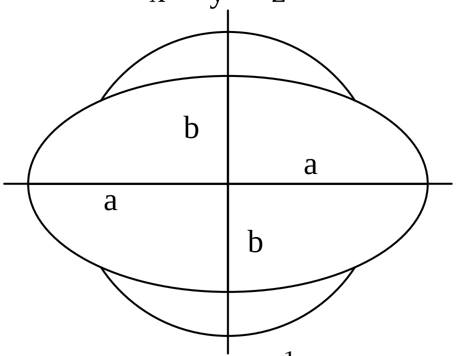
(4) Proof of Pythagoras theorem from the last theorem of P. Fermat--Part IV

(5) Series of p^n--Part V

(6) Solution of Simple Equations of Higher Powers by a New Method (Conversion Method)--Part VI

(7) Age of a Living Body on Other Planets & Relativity--Part VII

$x^n + y^n = z^n$



$$a^2 = b^2[4^{\{1-\frac{1}{n}\}} - 1]$$

Prof. Ramesh Chandra Ray
Electrical & Electronic Engineering
Chittagong University of Engineering & Technology
(CUET) Chittagong, Bangladesh
E-mail: ramesh_ray2003@yahoo.com
E-mail: rameshchandraray@yahoo.com

January, 2007

LAST THEOREM OF P. FERMAT

(with other topics)

www.rameshcray.wordpress.com

Prof. Ramesh Chandra Ray
Electrical & Electronic Engineering
Chittagong University of Engineering & Technology
(CUET) Chittagong, Bangladesh
E-mail: ramesh_ray2003@yahoo.com
E-mail: rameshchandraray@yahoo.com

January, 2007

[The solutions have come out at my age of sixty four(64) years.]

About the author

The author, son of Digamber Chandra Ray and Abeshmati Ray was born in the 3rd week of January, 1943 (1st week of Bengali month 'Magha' and year 1349) at the village Barender of Narail district in Bangladesh . He earned graduate degree in Electrical Engineering from BUET, Dhaka, in 1964 and M. Sc. in Electrical Engineering from the same institution in 1967.

Price: Taka-20/- (Twenty) only

Printed at: Kakali Press
Ahsan Ahmed Road
Khulna, Bangladesh

Preface

From the ancient age the mathematicians have been developing many numbers of theorems with their solutions. The subject matter of mathematics is vast and many authors have discussed the problems of various branches of mathematics into qualitative books. In this small book, simple elementary proof of the last theorem of P.Fermat, the famous French mathematician, has been discussed by algebraic and geometrical methods. A new concept of compressed circle has also been explained and a new method of proof of Pythagoras theorem from an unequal triangle has been stated. Moreover, a new process of solution of simple equations of higher powers has been elaborated. Also a common technique of developing series of numbers of higher powers has been shown. In the last part, age of a living body on other planets is calculated in the light of theory of relativity. It is of Secondary School and Higher Secondary Certificate level book. I hope the readers will get interest to go through the book.

I wish to express my acknowledgement and give thanks to them who encouraged me to prepare the book and spent their valuable time to review it for publication.

Author

Contents

CONSECRATION

With the memory of my parents, the book is dedicated to them.

PART-I

ALGEBRAIC PROOF OF LAST THEOREM OF P. FERMAT (1601-1665 AD)

Introduction: An elementary proof of the last theorem of P. Fermat, the famous French Mathematician, $x^n+y^n=z^n$, has been discussed by algebraic method. He invented and solved many theorems in his life time (1601-1665 AD). At the last part of his lifetime, he proposed this theorem, that is known as "Fermat's Last Theorem (FLT)". He did not show any solution of this theorem. It comes to know that he has written comment in the margin of a book -- "I have got a fine solution of this problem, but due to want of space in the margin, I cannot state there".

Theorem: The equation, $x^n+y^n=z^n$ has no positive integral solution for n>2.

Solution: Let, $r^n=p=(uy)/b$, $s^n=q=(vy)/c$, u>v, b>c, p>q, r>s.
Again let, u=mv, c=gb, m>g. Then p=(mvy)/b, q=(vy)/(gb).
Also let, $z=(r^n+s^n)=(p+q)$, $x=(r^n-s^n)=(p-q)$. So we get, $s^n=q=(z-x)/2$, $mg=(p/q)=(r^n/s^n)=(z+x)/(z-x)$.
Then we can derive,

$$y^n=(z^n-x^n)=\{(p+q)^n-(p-q)^n\}=\{(\frac{mvy}{b}+\frac{vy}{gb})^n-(\frac{mvy}{b}-\frac{vy}{gb})^n\},$$

or, $y^n=q^n\{(mg+1)^n-(mg-1)^n\}$.
Let, $\{(mg+1)^n-(mg-1)^n\}=(mg\pm k)^n$, where $k\neq 1$.
Then we get, $y^n=q^n(mg\pm k)^n$, or,
$2^ny^n=(z+x+kx-kz)^n$, ………. (by putting the values of q and mg)
$2^ny^n=(z+x+kz-kx)^n$, ………. (by putting the values of q and mg)
or, $(z+x+kx-kz)^n+2^nx^n-2^nz^n=f(x,z)$………….. (1)
also, $(z+x+kz-kx)^n+2^nx^n-2^nz^n=f(x,z)$………….. (1a)

To obtain the minimum value of the equations (1), (1a), we take partial differentiation with respect to x, z and considering the equations are equal to 0, we obtain,

$(k-1)x^{n-1}=(k+1)z^{n-1}$, or, $x(k-1)^{1/(n-1)}=z(k+1)^{1/(n-1)}$

and also $(k+1)x^{n-1}=(k-1)z^{n-1}$, or, $x(k+1)^{1/(n-1)}=z(k-1)^{1/(n-1)}$.

Putting the value of z at the equation (1), (1a), we get,

$x^n[\{1+k+(1-k)(\frac{k-1}{k+1})^{1/(n-1)}\}^n-2^n(\frac{k-1}{k+1})^{n/(n-1)}+2^n]=f_{\min}$ .... (2)

$x^n[\{1-k+(1+k)(\frac{k+1}{k-1})^{1/(n-1)}\}^n-2^n(\frac{k+1}{k-1})^{n/(n-1)}+2^n]=f_{\min}$ .... (2a)

Again putting the value of x at the equation (1), (1a), we get,

$z^n[\{1-k+(1+k)(\frac{k+1}{k-1})^{1/(n-1)}\}^n+2^n(\frac{k+1}{k-1})^{n/(n-1)}-2^n]=f_{\min}$ .... (3)

$z^n[\{1+k+(1-k)(\frac{k-1}{k+1})^{1/(n-1)}\}^n+2^n(\frac{k-1}{k+1})^{n/(n-1)}-2^n]=f_{\min}$ .... (3a)

So k will have no rational value from the equations (2), (2a), (3), (3a) to obtain 0 value of $f_{\min}$ when n>2.

[Example: Let k=(13/12), n=3. Then, $x^3\{(992/125)+(31/15)^3\}=f_{\min}$ and also $z^3\{992+(31/3)^3\}=f_{\min}$]

For n=2, we get $f_{\min}=0$ when k=0. Then, x=3, y=4, z=5.

For n=2, we get from the equations (1), (1a) when $f(x,z)=0$,

$(z+x+kx-kz)^2+4x^2-4z^2=0$ and

$(z+x+kz-kx)^2+4x^2-4z^2=0$, or, $\pm k=(\frac{z+x}{z-x})-2(\frac{z+x}{z-x})^{1/2}$.

[Examples:- (i) x=3, y=4, z=5, k=0.(ii) x=20, y=21, z=29, k=7/9.]

Hence *x, y, z, n* will have no positive integral solution for *n>2*.

It is completing the proof of Last Theorem of P. Fermat by algebraic method.

PART-II

GEOMETRICAL PROOF OF LAST THEOREM OF P.FERMAT (1601-1665 AD)

Introduction: An elementary proof of the last theorem of the French Mathematician P. Fermat, $x^n + y^n = z_n^{\ n}$, has been discussed by geometrical method. For n = 2, the equation $x^2 + y^2 = z_2^{\ 2}$, represents right angle triangle. For any value of n, it is obtained that $z_{n+2} < z_{n+1} < z_n < z_{n-1} < z_{n-2} ...... < z_2$, or $z_2 > z_3 > z_4 > z_5$…etc(proof is given below). These are shown in fig.1(a) and fig.1(b).

[Example:- $20^2 + 21^2 = 29^2$. Then, $20^3 + 21^3 = 17261 < 29^3$.]

Theorem: In the equation, $x^n + y^n = z_n^{\ n}$, there is no solution when x, y, z_n and n > 2 are positive integers.

Proof of $z_{n+2} < z_{n+1} < z_n < z_{n-1} < z_{n-2} ... < z_2$, or $z_2 > z_3 > z_4 > z_5$….etc(stated above):

Let, $z_2 = (x^2 + y^2)^{1/2}$, $z_3 = (x^3 + y^3)^{1/3}$,….$z_n = (x^n + y^n)^{1/n}$, $z_{n+r} = (x^{n+r} + y^{n+r})^{1/(n+r)}$, $z_{n+s} = (x^{n+s} + y^{n+s})^{1/(n+s)}$, r > s and $z_{n+r} < z_{n+s}$, or,$(x^{n+r} + y^{n+r})^{1/(n+r)} < (x^{n+s} + y^{n+s})^{1/(n+s)}$.

Then $(x^{n+r} + y^{n+r})^{(n+s)} < (x^{n+s} + y^{n+s})^{(n+r)}$…………(a)

As x < y or y < x or y = x, let x = my or y = mx and m < 1 or m = 1. Then it is obtained from (a) that,

$$(1 + m^{n+r})^{(n+s)} < (1 + m^{n+s})^{(n+r)}, \text{ or,}$$

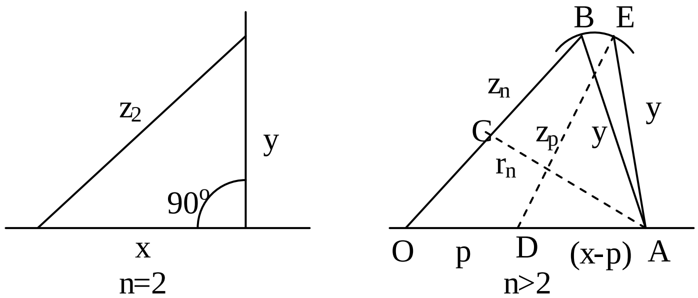
$$0 < \{ {}^{(n+r)}C_1 m^s - {}^{(n+s)}C_1 m^r \} + m^n \{ {}^{(n+r)}C_2 m^{2s} - {}^{(n+s)}C_2 m^{2r} \} + m^{2n} \{ {}^{(n+r)}C_3 m^{3s} - {}^{(n+s)}C_3 m^{3r} \} + ... (b)$$

So it is obtained from (b) for m < 1 or m = 1 and any value of n, r, s, r > s that, LHS < RHS.

Hence, $z_{n+2} < z_{n+1} < z_n < z_{n-1} < ... < z_2$, or $z_2 > z_3 > z_4 > z_5$…etc.

Proof of the theorem: As it is proved that $z_{n+2} < z_{n+1} < z_n < z_{n-1} <$ …$< z_2$, or $z_2 > z_3 > z_4 > z_5$ ...etc, then the following fig.1(a) and fig.1(b) can be drawn. Let OA= x, OD = p, DA = (x - p), arc BE, AB = AE = y, OB = z_n, OC = CB = $z_n/2$, DE = z_p,

$CA = r_n$ and the angles $\angle OAB < 90^o$, $\angle DAE < 90^o$ (for $n > 2$). Considering various values of p, it can be represented



n=2
(a)
Fig.1
n>2
(b)

in accordance with the Fermat's theorem, $x^n + y^n = z_n^{\,n}$ from the triangle ΔOAB and also $(x - p)^n + y^n = z_p^{\,n}$ from the triangle ΔDAE.

Then $z_p < z_n$ and $z_p > y$. By applying Apollonius theorem to the triangle ΔOAB, it is obtained that,

$$x^2 + y^2 = 2r_n^{\,2} + z_n^{\,2}/2 = z_2^{\,2}\ldots\ldots\ldots\ldots(i)$$

Again by using 'square rule', $[A^2 + B^2 = C^2, A = (R^2 - S^2), B = 2RS, C = (R^2 + S^2), R > S]$, the following relations can be written from (i): $x = (R_1^{\,2} - S_1^{\,2})$, $y = 2R_1S_1$, $R_1 > S_1$, $z_n = (R_2^{\,2} - S_2^{\,2})\surd 2$, $r_n\surd 2 = 2R_2S_2$, $R_2 > S_2$, $z_2 = (R_1^{\,2} + S_1^{\,2}) = (R_2^{\,2} + S_2^{\,2})$, or $(R_1^{\,2} - R_2^{\,2}) = (S_2^{\,2} - S_1^{\,2}) = g$,
or $R_1^{\,2} = (R_2^{\,2} + g)$ and $S_1^{\,2} = (S_2^{\,2} - g)$.

Then $x = (R_1^{\,2} - S_1^{\,2}) = (R_2^{\,2} - S_2^{\,2}) + 2g = 2g + z_n/\surd 2,\ldots\ldots(1)$

or, $2g = x - z_n/\surd 2\ldots\ldots\ldots\ldots(1)$

Putting this value of (1), it is obtained that

$$(2g + z_n/\surd 2)^n + y^n = z_n^{\,n}\ldots\ldots\ldots\ldots(2)$$

So it is determined by the value of 2g from (1) and (2) whether x, y, n and z_n will be simultaneously integers or not. To obtain the possible integral values of x and z_n from (1), there may be a value of $2g = (p - q/\sqrt{2})$, where p and q are integers. Then it is obtained from the equation (1) that,

$$(x - p) = (z_n - q)/\sqrt{2} \ldots\ldots\ldots\ldots(ii)$$

$$(p \rightarrow x,\ z_n \rightarrow z_p,\ z_p \rightarrow y),\ (p = x,\ z_n = z_p = y = q)$$

When $p = x$ and $q = z_n$ at (ii), then $(x - p)^n + y^n = z_p^{\ n}$, or $0 + y^n = z_p^{\ n}$, or, $z_p = y$. But $z_p > y$ for validity of Fermat's equations.

So $x > p$ and $z_n > q$.Then z_n will not be integer from (ii) for $x > p$.

[Example:- Let $x = 3$, $y = 5$ and $p = 1, 2, 3$. Then it can be written, $3^n + 5^n = z_n^{\ n}$ and $(3 - p)^n + 5^n = z_p^{\ n}$.

For $p = 1$, the Fermat's equation, $(3 - 1)^n + 5^n = z_p^{\ n}$,

or, $2^n + 5^n = z_p^{\ n}$ is valid, $z_p > y$.

For $p = 2$, the Fermat's equation, $(3 - 2)^n + 5^n = z_p^{\ n}$,

or, $1^n + 5^n = z_p^{\ n}$ is valid, $z_p > y$.

For $p = 3$, the Fermat's equation, $(3 - 3)^n + 5^n = z_p^{\ n}$,

or, $0 + 5^n = z_p^{\ n}$ is invalid. Then $z_p = 5 = y$.

Again let $(x - p) = 3, 4, 5,\ldots$.etc. Then $3^n + 5^n = z_p^{\ n}$,

$4^n + 5^n = z_p^{\ n}$, $5^n + 5^n = z_p^{\ n}$, $6^n + 5^n = z_p^{\ n}$,....etc.

So $x > p$ and $z_n > z_p > y$ for validity of the Fermat's equations.]

Hence x or y or z_n or $n > 2$ is not integer from the equations (1) and (2) for any value of 2g.

It proves the last theorem of P. Fermat by geometrical method.

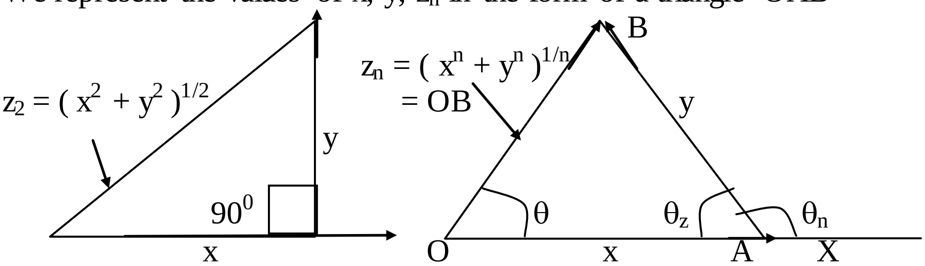
PART-III

COMPRESSED CIRCLE

Introduction: Generally we are familiar with the conics like circle, ellipse, parabola, hyperbola etc. When circle is compressed diametrically in opposite direction, the figure shows like an ellipse. But it has been proved here that actually it is not ellipse. The general equations of compressed circle are mathematically derived from the last theorem of P. Fermat. For n = 2, it is circle, $x^2 + y^2 = z_2^{\,2}$. For n > 2, the equations describe the concept of compressed circles.

Derivation of compressed circle:

Let us take the equation $z_n^{\,n} = x^n + y^n$, where $z_n = (x^n + y^n)^{1/n}$. We represent the values of x, y, z_n in the form of a triangle OAB



n = 2 Fig.1 n > 2

as shown in Fig.1 above, where OA = x, AB = y, OB = z_n and the angles, $\angle AOB = \theta$, $\angle OAB = \theta_z$, $\angle BAX = \theta_n$, $(\theta_z + \theta_n) = 180^o$. Then we get from the triangle OAB for any value of x, y, z_n that,

$$Cos\theta_Z = \frac{x^2 + y^2 - (x^n + y^n)^{\frac{2}{n}}}{2xy}, or, Cos\theta_z = \frac{N}{2xy} = \frac{z_2^{\,2} - z_n^{\,2}}{2xy},$$

where $N = \{(x^2 + y^2) - (x^n + y^n)^{2/n}\}$. So when n = 2, then N = 0, $\cos\theta_z = 0$, or, $\theta_z = 90^0$ and $\theta_n = 180^0 - 90^0 = 90^0$ (see fig.1).

$$\text{So, } z_n < z_2 \text{ and } \theta_z = \cos^{-1}\frac{[x^2 + y^2 - (x^n + y^n)^{\frac{2}{n}}]}{2xy} \quad ....(1)$$

Also we can derive from fig.1, $\sin^n(\theta_z + \theta) = \sin^n\theta_z - \sin^n\Theta$.....(2)

For n = 1, $\theta = 0^o$, $\theta_n = 0^o$, $\theta_z = 180^o$. For n = 2, $\theta_z = \theta_n = 90^o$, $\sin^2\theta + \cos^2\theta = 1$. So it is obtained from (1) and (2) for $n > 2$, $\theta_z < 90^o$, $\theta_n = (180^0 - \theta_z)$ and $\theta_n > 90^o$.

As $(x^2 + y^2) = z_2^2$ represents circle, $z_n < z_2$ and $\theta_z < 90^o$, so the circle is compressed diametrically in opposite direction along the v-axis. These are shown below in Fig.2(i) and Fig.2(ii) of circle

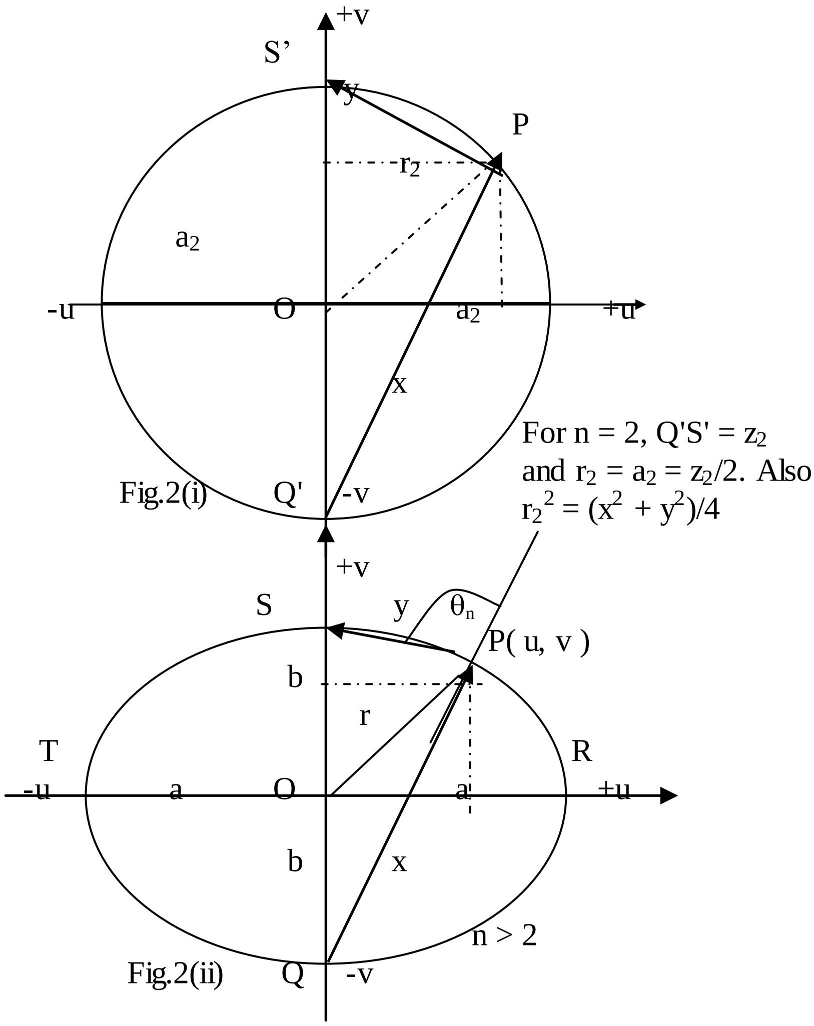


Fig.2(i)

Fig.2(ii)

and compressed circle(not ellipse) respectively. Then the locus of any point P(u, v), the co-ordinates of which are (u, v), will be QRSTQ considering O as the origin. -u to +u and -v to +v are two co-ordinate axes. Then (x, y) can be transformed to (u, v) and vice-versa. Let, OR = OT = a, OS = OQ = b, QP = x, PS = y, QS = z_n = 2b, OP = r (for n > 2). Then we get,

$x^n + y^n = z_n{}^n = (2b)^n$.............(3), $u^2 + (b + v)^2 = x^2$............(4)

and $u^2 + (b - v)^2 = y^2$...............(5)

Subtracting (5) from (4) we get, $4bv = x^2 - y^2$, or,

$$v = \frac{(x^2 - y^2)}{4b} \text{.........................} (6)$$

Adding (4) and (5) we get,

$$u^2 + v^2 = \frac{(x^2 + y^2)}{2} - b^2 = \frac{(x^2 + y^2)}{2} - \frac{z_n{}^2}{4} \text{.............} (7)$$

{Note:- We get from (7), $(x^2 + y^2) = 2(u^2 + v^2) + 2b^2 = 2b^2 + 2r^2$. So, $(QP)^2 + (PS)^2 = 2(OS)^2 + 2(OP)^2$, is the theorem of the Greek mathematician, Apollonius, for any triangle.}

$$\text{or, } u^2 + v^2 = \frac{(x^2 + y^2)}{2} - \frac{(x^n + y^n)^{2/n}}{4}.$$

Putting the values of x and y from (4) and (5) at (3), we get

$[u^2 + (b - v)^2]^{n/2} + [u^2 + (b + v)^2]^{n/2} = (2b)^n$, ...(8)

which is also the general equation of compressed circle in terms of u, v, b, n and the compressed diameter z_n = 2b, shown in Fig.2(ii). Then we get the slope of compressed circle of Fig.2(ii),

$$\frac{dv}{du} = \frac{u[u^2 + (b-v)^2]^{(n-2)/2} + u[u^2 + (b+v)^2]^{(n-2)/2}}{(b-v)[u^2 + (b-v)^2]^{(n-2)/2} - (b+v)[u^2 + (b+v)^2]^{(n-2)/2}}$$

and we get the slope of circle of Fig.2(i), $\frac{dv}{du} = -\frac{u}{v}$.

Now when x = y, then, $x^2 = a^2 + b^2$ and $y^2 = a^2 + b^2$, or, $x^n = (a^2 + b^2)^{n/2}$ and $y^n = (a^2 + b^2)^{n/2}$. Then we get from (3),

$(a^2 + b^2)^{n/2} + (a^2 + b^2)^{n/2} = (2b)^n$, or, $a^2 = b^2\{4^{(1-\frac{1}{n})} - 1\}$ …….(9)

Also we can derive (9) from (8) when u = a and v = 0. Considering these values of a and b, the equation of an ellipse will be, $\frac{u^2}{a^2} + \frac{v^2}{b^2} = 1$, or, $\frac{u^2}{b^2\{4^{(1-\frac{1}{n})} - 1\}} + \frac{v^2}{b^2} = 1$

and the slope of ellipse, $\frac{dv}{du} = \frac{-u}{v\{4^{(1-\frac{1}{n})} - 1\}}$.

Hence the slope of ellipse is not same as the slope of compressed circle. When n = 3, 4, 5 …….etc, we get from (9), $a^2 \rightarrow 3b^2$. For $n = \infty$, $a = b\sqrt{3}$, $\theta_z = 60^o$ and $x = y = z_n = 2b$. For n = 2, we get a = b, that is for circle. Then we get from (7) for n > 2,

$u^2 + v^2 = r^2 = (x^2 + y^2)/2 - b^2$,

or, $x^2 + y^2 = 2b^2 + 2r^2 = z_n^2 + 2r^2$,…………..(10)

where $r^2 = b^2 \rightarrow 3b^2$ for n = 2, 3, 4,….. etc. When n = 2, $x^2 + y^2 = 4b^2 = (2b)^2 = (2a)^2 = z_2^2$. So we get, $r^2 = z_n^2/4 + xy \cos\theta_z = b^2 + xy \cos\theta_z$, or, $\cos\theta_z = \frac{(r^2 - b^2)}{xy}$ .... (11)

So we get from (11), $\theta_z = 90^o$, when r = b for circle. Therefore, (3), (7), (8) and (10) are the equations of compressed circles. The values of u, v, r, y, Θ, Θ_z, Θ_n can be calculated when the magnitude values of b, x, n are known and x < 2b or y < 2b.

[Examples:- (a) Draw the shape of compressed circle for 1 < n < 2.

(b) The cross-section through the center and north-south end points of earth may be an example, where the diameter (2b) through the north-south end points is compressed.

(c) Another example is cited.

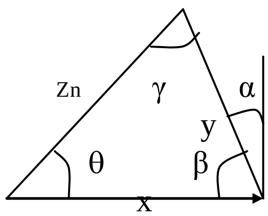
When an spherical hollow bladder of football is rotated at an angular velocity about its central axis, its shape will be similar to the earth.

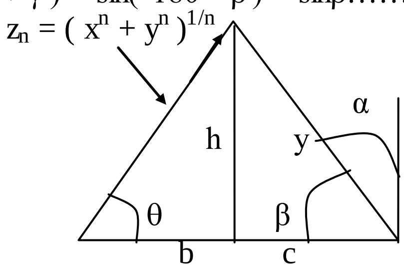
(d) There may be an example in metal cutting.]

PART-IV

PROOF OF PYTHAGORAS THEOREM FROM THE LAST THEOREM OF P. FERMAT

Considering a triangle of which $z_n > x$, $z_n > y$, $\beta < 90^o$, $(\theta + \beta + \gamma) = 180^o$, $x^n + y^n = z_n^n$ and $(\alpha + \beta) = 90^o$, let us draw the perpendicular h upon x so that $x = (b + c)$. Then the following relations are obtained: $c = y\cos\beta$, $h = y\sin\beta$, $b = z_n\cos\theta$, $h = z_n\sin\theta$, $c = y\sin\alpha$, $h = y\cos\alpha$, $b = z_n\sin(\gamma - \alpha)$, $h = z_n\cos(\gamma - \alpha)$ and $\sin(\theta + \gamma) = \sin(180^o - \beta) = \sin\beta$……..(i)





By rearrangement, it can be derived that,

$$x = (b + c) = z_n\cos\theta + y\cos\beta = z_n\sin(\gamma - \alpha) + y\sin\alpha,$$

$$x = z_n\sin(\theta + \beta)/\sin\beta, \quad y = z_n\sin\theta/\sin\beta.$$

By considering the higher powers, the following general expressions are derived for any value of n:

$$x^n = \{z_n\sin(\theta + \beta)/\sin\beta\}^n, \quad y^n = (z_n\sin\theta/\sin\beta)^n,$$

$$(x - y\cos\beta)^n + (y\sin\beta)^n = z_n^n(\sin^n\theta + \cos^n\theta),$$

$$\sin^n\theta + \cos^n(\theta + \beta - 90^o) = \cos^n(90^o - \beta) = \cos^n\alpha$$

and from (i), $\sin\theta\cos\gamma + \cos\theta\sin\gamma = \sin\beta$.

For the particular values of $n = 2$, $\beta = 90^o$ and $\alpha = 0^o$, it is obtained that, $c = 0$, $b = x$, $h = y$, $x = z_n\cos\theta$, $x = z_n\sin\gamma$, $y = z_n\sin\theta$, $y = z_n\cos\gamma$, $\sin^2\theta + \cos^2\theta = 1$ and $x^2 + y^2 = z_2^2$.

So the Pythagoras theorem is the special condition when $n = 2$ and $\beta = 90^o$ of the last theorem of P. Fermat and the triangle of which $z_n > x$, $z_n > y$, $\beta < 90^o$, $(\theta + \beta + \gamma) = 180^o$ and $x^n + y^n = z_n^n$ for any value of n.

PART-V

SERIES OF $\mathbf{p^n}$

Introduction: A process has been explained to develop the series of $p^2, p^3, p^4, \ldots p^n$. The series are developed gradually from lower power to higher power by a common process. Several characteristics of the series are also stated.

Series of $\mathbf{p^n}$: The series can be written as follows:

$p^n = 1 + (1 + c_1) + (1 + c_1 + c_2) + (1 + c_1 + c_2 + c_3) + \ldots + p_{th}$,

or, $p^n = p + (p - 1)c_1 + (p - 2)c_2 + (p - 3)c_3 + \ldots\ldots + p_{th}$

The values of $c_1, c_2, c_3, \ldots$etc, can be found out from

$$c_r = 2[\frac{n(n-1)}{!2} r^{(n-2)} + \frac{n(n-1)(n-2)(n-3)}{!4} r^{(n-4)} + \ldots],$$

or, $c_r = (r + 1)^n + (r - 1)^n - 2r^n$, where $r = 1, 2, 3 \ldots\ldots$etc

and $c_1 < c_2 < c_3 \ldots\ldots$etc, if $n > 2$.

From p^2 we can develop a series,

$$1 + 2 + 3 + \ldots\ldots + p = p(p + 1)/2$$

and from p^3 we can develop a series,

$$1 + 2^2 + 3^2 + 4^2 + \ldots\ldots + p^2 = p(p + 1)(2p + 1)/6$$

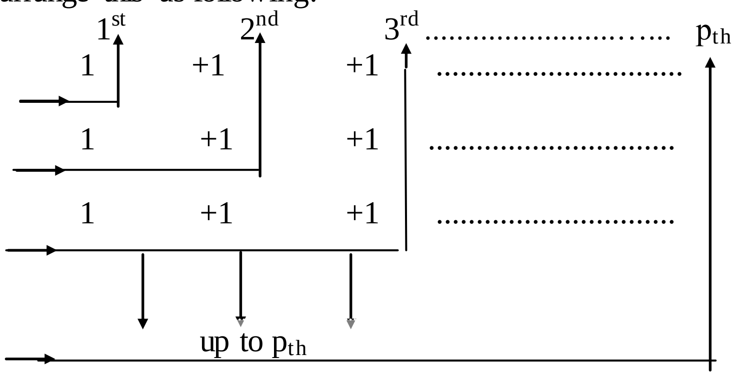
Again from p^4 we can develop a series,

$$1 + 2^3 + 3^3 + 4^3 + \ldots\ldots + p^3 = \{p(p + 1)/2\}^2$$

(A) Square of numbers

$p^2 = p \times p = (1 + 1 + 1 + 1 + \ldots\ p_{th})\mathrm{x}(1 + 1 + 1 + 1 + \ldots\ p_{th})$

We can arrange this as following:



We start adding the numbers from left as underlined by the arrow marks to obtain the series of summation from lower value to gradual higher value of terms of

$p^2 = 1 + 3 + 5 + 7 + 9 + 11 + \ldots\ldots + p_{th}$ (odd numbers only).

We observe several characteristics in the series as follows:

(1) The difference between the values of the consecutive terms is equal and is always a constant value, c = 2.

(2) The value of any r_{th} term of the series of p^2 will be

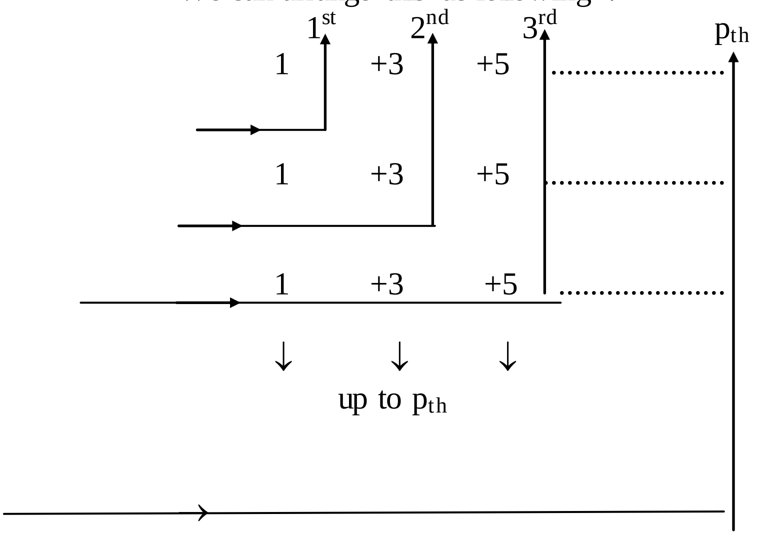
$R_r = r^2 - (r - 1)^2$, where r = 1, 2 ,3, 4,...................etc.

(B) Cube of numbers

$p^3 = p^2 \times p$, or, $p^3 = (1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + \ldots.\ p_{th})\times$

$(1 + 1 + 1 + 1 + \ldots\ldots\ldots\ p_{th})$.

We can arrange this as following :



We start adding the numbers from left as underlined by arrow marks to obtain the series of summation from lower value to gradual higher value of terms of

$p^3 = 1 + 7 + 19 + 37 + 61 + 91 + 127 + 169 + ... + p_{th}$

This series can also be written by another suitable form as follows: $p^3 = 1 + \{1 + 3c\} + \{1 + (1 + 2) \times 3c\} + \{1 + (1 + 2 + 3) \times 3c\} + \{1 + (1 + 2 + 3 + 4) \times 3c\} + ... + p_{th}$,

where c = 2, a characteristic of the series of p^2. We observe several characteristics in the series as follows:

(1) The difference between the values of the consecutive terms is not equal and increases gradually step by step 3c, 2x3c, 3x3c, 4x3c, 5x3c and so on.

(2) The value of any r_{th} term of the series of p^3 will be

$R_r = r^3 - (r - 1)^3$, where r = 1 , 2, 3, 4, ...etc.

(C) Higher powers of numbers

$p^4 = p^3 \times p$, $p^5 = p^4 \times p$,........................ $\longrightarrow$ p^n =............

By the above process we get series of summation from lower value to gradual higher values of terms. Then,

$p^4 = 1 + 15 + 65 + 175 + 369 + 671 + 1105 + 1695 +$......upto p_{th},

$p^5 = 1+31+211+781+2101+4651+9031+15961+$.......upto p_{th} ,

....

$p^n = 1 + (2^n - 1) + (3^n - 2^n) + (4^n - 3^n) +$........ upto p_{th}.

So any series of p^n can be developed from the series of p.

We observe several characteristics in all the series as follows:

(1) The difference between the values of the consecutive terms is not equal and increases rapidly.

(2) The value of any r_{th} term of a series of p^n will be

$R_r = r^n - (r - 1)^n$, where r = 1, 2, 3, 4,.............etc.

(D) Summation of Combination

Let, $(r + s)^n = {}^nC_0r^n + {}^nC_1r^{n-1}s + {}^nC_2r^{n-2}s^2 + {}^nC_3r^{n-3}s^3 + \ldots$,

$(r + s)^n + (r - s)^n = 2({}^nC_0r^n + {}^nC_2r^{n-2}s^2 + {}^nC_4r^{n-4}s^4 + \ldots.)$,

$(r + s)^n - (r - s)^n = 2({}^nC_1r^{n-1}s + {}^nC_3r^{n-3}s^3 + {}^nC_5r^{n-5}s^5 + \ldots..)$.

For r = s, it will be $2^n = {}^nC_0 + {}^nC_1 + {}^nC_2 + {}^nC_3 + {}^nC_4 + \ldots...$

and ${}^nC_0 + {}^nC_2 + {}^nC_4 + \ldots... = {}^nC_1 + {}^nC_3 + {}^nC_5 + \ldots...$

PART-VI

SOLUTION OF SIMPLE EQUATIONS OF HIGHER POWERS BY A NEW METHOD (CONVERSION METHOD)

Introduction: In this section a new method of solution of simple equations of higher powers has been explained. The method may be said 'Conversion Method'.

(A) Let us take an equation,

$$y = (x + a)(x + b) = x^2 + (a + b)x + ab = x^2 + Px + Q$$

To solve the equation for x when y = 0, we adopt the following steps: (1) Find $\frac{dy}{dx}$ = 0 and then put x = X, an average value of the roots to obtain $X = -\frac{P}{2} = -\frac{a+b}{2}$, or, P = - 2X

(2) Put the value of P in the equation and find the value of

$$y = Y = X^2 + PX + Q = Q - X^2$$

If Y = 0 then X is one root of the equation.

(3) Find the values of the roots (- a) = X - d_1 and (- b) = X - d_2, where d_1 and d_2 are termed as deviations from the average value of the roots. To find (- a) and (- b), put

(i) $d_1 + d_2 = 0$,

(ii) $d_1 d_2 = Y = Q - X^2$

Then we get $d_1, d_2 = \pm\sqrt{(-Y)} = \pm\sqrt{X^2 - Q}$. So,

$$(- a), (- b) = X \pm \sqrt{X^2 - Q}.$$

[Hints: To prove (3)(i),(ii) we take, - (a + b) = - P = (2X - d_1 - d_2) and ab = Q = (X - d_1)(X - d_2).]

Example: Let, $y = x^2 + 6x + 25 = x^2 + Px + Q$.

Then, $\frac{dy}{dx} = 2x + 6$. So, 2X + 6 = 0, or, X = - 3.

So the roots of the equation will be

$$(- a), (- b) = X \pm\sqrt{X^2 - Q} = -3 \pm\sqrt{9 - 25} = -3 \pm j4$$

(B) Let us take an equation, $y = (x + a)(x + b)(x + c) = x^3 + (a + b + c)x^2 + (ab + bc + ca)x + abc = x^3 + Px^2 + Qx + R$.
To solve the equation for x when $y = 0$, we adopt the following steps: (1) Find $\frac{d^2y}{dx^2} = 0$ and then put $x = X$, an average value of the roots to obtain $X = -\frac{P}{3} = -\frac{a+b+c}{3}$, or, $P = -3X$

(2) Put the value of P in the equation and find the value of

$$y = Y = X^3 + PX^2 + QX + R = R + QX - 2X^3$$

If $Y = 0$, then X is one root of the equation.

(3) Find the values of the roots $(-a) = X - d_1$, $(-b) = X - d_2$ and $(-c) = X - d_3$, where d_1, d_2, d_3 are termed as deviations from the average value of the roots. To find $(-a)$, $(-b)$ and $(-c)$, put

(i) $d_1 + d_2 + d_3 = 0$,
(ii) $d_1d_2 + d_2d_3 + d_3d_1 = Q - 3X^2$,
(iii) $d_1d_2d_3 = Y = R + QX - 2X^3$

[Hints : To prove (3)(i),(ii),(iii) we take
$-(a + b + c) = -P = (3X - d_1 - d_2 - d_3)$,
$(ab + bc + ca) = Q = (X - d_1)(X - d_2) + (X - d_2)(X - d_3) + (X - d_3)(X - d_1)$ and
$-abc = -R = (X - d_1)(X - d_2)(X - d_3)$.]

(4) To solve for d_1, d_2, d_3, let $d_1 = A$, $d_2 = (B + jC)$, $d_3 = (B - jC)$. Then we get,

$$d_1 + d_2 + d_3 = A + 2B = 0. \text{ So, } B = -A/2.$$

Again, $d_1d_2 + d_2d_3 + d_3d_1 = B^2 + C^2 + 2AB = (Q - 3X^2)$,

$$\text{or, } C = \pm\sqrt{\frac{3A^2}{4} + (Q - 3X^2)}.$$

Again , $d_1d_2d_3 = A(B^2 + C^2) = Y = (R + QX - 2X^3)$.
Then we get , $A^3 + (Q - 3X^2)A - (R + QX - 2X^3) = 0$.

This is the form as following :

$$p^3 + Mp + N = 0, \text{ where } p = A.$$

(i) To solve this equation we factorize the value of N so that mn = N. Then we use the values of m and n to get,

$$p^2 + M + \frac{N}{P} = 0, \qquad m^2 + n = - M, \qquad n = \frac{N}{p}$$

Hence m may be one real root of the above equation, that is, A = m or A = - m. Then we can find the values of B and C. Hence the roots (- a), (- b), (- c) can be found out.

(ii) When the factors m and n are not found out as integers by the process described above or if these are fractions but not easily available, then we can use the following to find the value of one real root of p or A:

$p = A = g^{1/3} + h^{1/3}$, $g + h + N = 0$, $3(gh)^{1/3} + M = 0$, so that, $p = A$

$$= \left[-\frac{N}{2} + \sqrt{\left\{\frac{N}{2}\right\}^2 + \left\{\frac{M}{3}\right\}^3} \right]^{1/3} + \left[-\frac{N}{2} - \sqrt{\left\{\frac{N}{2}\right\}^2 + \left\{\frac{M}{3}\right\}^3} \right]^{1/3}$$

$$= (r + j\sqrt{s}) + (r - j\sqrt{s}), \quad \text{so that} \quad p = A = 2r.$$

Example: Let, $y = x^3 + 8x^2 + 37x + 50 = x^3 + Px^2 + Qx + R$.

Then, $\frac{d^2y}{dx^2} = 6x + 16$. So, $6X + 16 = 0$, or, $X = -\frac{8}{3}$.

Then, $(Q - 3X^2) = 37 - 3\left[\frac{8}{3}\right]^2 = \frac{47}{3}$ and

$$y = Y = R + QX - 2X^3 = 50 - 37\left[\frac{8}{3}\right] + 2\left[\frac{8}{3}\right]^3 = -\frac{290}{27}.$$

So we get, $A^3 + \frac{47}{3}A + \frac{290}{27} = 0.$

By applying the procedure described before, we get one real root of the above equation, $A = -\frac{2}{3}$, so that

$$A^2(A+\frac{2}{3}) - \frac{2A}{3}(A+\frac{2}{3}) + \frac{145}{9}(A+\frac{2}{3}) = 0.$$

So, $d_1 = -\frac{2}{3}$, $d_2 = \frac{1}{3} + j4$, and $d_3 = \frac{1}{3} - j4$. Hence the roots will be $-a = X - d_1 = -\frac{8}{3} + \frac{2}{3} = -2,$

$$-b = X - d_2 = -\frac{8}{3} - \frac{1}{3} - j4 = -(3+j4), -c = X - d_3 = -\frac{8}{3} - \frac{1}{3} + j4 = -(3-j4)$$

(C) Let us take an equation, $y = (x + a)(x + b)(x + c)(x + e)$
$= x^4 + (a + b + c + e)x^3 + (ab + ac + ae + bc + be + ce)x^2 +$
$(abc + abe + ace + bce)x + abce = x^4 + Px^3 + Qx^2 + Rx + S.$

To solve the equation for x when $y = 0$, we adopt the following steps: (1) Find $\frac{d^3y}{dx^3} = 0$ and then put $x = X$, an average value of the roots to obtain $X = -\frac{P}{4} = -\frac{a+b+c+e}{4}$, or, $P = -4X$

(2) Put the value of P in the equation and find the value of

$$y = Y = x^4 + Px^3 + Qx^2 + Rx + S = S + RX + QX^2 - 3X^4$$

If $Y = 0$, then X is one root of the equation.

(3) Find the values of the roots $(-a) = X - d_1$, $(-b) = X - d_2$, $(-c) = X - d_3$ and $(-e) = X - d_4$, where d_1, d_2, d_3, d_4 are termed as deviations from the average value of the roots. To find $(-a)$, $(-b)$, $(-c)$ and $(-e)$, put

(i) $d_1 + d_2 + d_3 + d_4 = 0,$

(ii) $d_1d_2 + d_1d_3 + d_1d_4 + d_2d_3 + d_2d_4 + d_3d_4 = Q - 6X^2,$

(iii) $d_1d_2d_3 + d_1d_2d_4 + d_1d_3d_4 + d_2d_3d_4 = R + 2QX - 8X^3,$

(iv) $d_1d_2d_3d_4 = Y = S + RX + QX^2 - 3X^4$

[Hints: To prove (3)(i),(ii),(iii),(iv) we take

$- (a + b + c + e) = - P = (4X - d_1 - d_2 - d_3 - d_4)$,

$(ab + ac + ae + bc + be + ce) = Q = (X - d_1)(X - d_2) + (X - d_1)(X - d_3) + (X - d_1)(X - d_4) + (X - d_2)(X - d_3) + (X - d_2)(X - d_4) + (X - d_3)(X - d_4)$,

$- (abc + abe + ace + bce) = - R = (X - d_1)(X - d_2)(X - d_3) + (X - d_1)(X - d_2)(X - d_4) + (X - d_1)(X - d_3)(X - d_4) + (X - d_2)(X - d_3)(X - d_4)$ and

$abce = S = (X - d_1)(X - d_2)(X - d_3)(X - d_4)$.]

(4) To solve for d_1, d_2, d_3, d_4, let $d_1 = (A + jB)$, $d_2 = (A - jB)$, $d_3 = (C + jD)$, $d_4 = (C - jD)$.Then we get, $d_1 + d_2 + d_3 + d_4 = 2(A + C) = 0$, or, $A = - C$.

Again, $d_1d_2 + d_1d_3 + d_1d_4 + d_2d_3 + d_2d_4 + d_3d_4 = A^2 + B^2 + C^2 + D^2 + 4AC = (Q - 6X^2)$.

Again, $d_1d_2d_3 + d_1d_2d_4 + d_1d_3d_4 + d_2d_3d_4 = 2C(A^2 + B^2) + 2A(C^2 + D^2) = (R + 2QX - 8X^3)$.

Again, $d_1d_2d_3d_4 = (A^2 + B^2)(C^2 + D^2) = Y = (S + RX + QX^2 - 3X^4)$. Then we get,

$$A^6 + \left[\frac{Q - 6X^2}{2}\right]A^4 + \left[\frac{(Q - 6X^2)^2}{16} - \frac{(S + RX + QX^2 - 3X^4)}{4}\right]A^2$$

$$- \frac{(R + 2QX - 8X^3)^2}{64} = 0.$$

It is the form as follows: $p^3 + Mp^2 + Np + K = 0$, where $p = A^2$.

So a positive and real value of p may be found out by applying the process described in (B). Example:- Let, $y = x^4 + 12x^3 + 86x^2 + 268x + 533 = x^4 + Px^3 + Qx^2 + Rx + S$.

Then, $\frac{d^3y}{dx^3} = 24x + 72$. So, $24X + 72 = 0$, or, $X = - 3$.

Then, $(Q - 6X^2) = 86 - 6(3)^2 = 32$, $(R + 2QX - 8X^3) = 268 – 2(86)(3) + 8(3)^3 = - 32$ and $y = Y = (S + RX + QX^2 - 3X^4) = 533 - 268(3) + 86(3)^2 - 3(3)^4 = 260$. So we get,

$A^6 + 16A^4 - A^2 - 16 = 0$, or, $p^3 + 16p^2 - p - 16 = 0$, where $p = A^2$.

By applying the procedure described before, we get one positive and real root of the above equation, $p = A^2 = 1$, so that

$$p^2(p - 1) + 17p(p -1) + 16(p - 1) = 0$$

Therefore, $A = \sqrt{1} = \pm 1$. So,

$d_1 = - 1 + j3$, $d_2 = - 1 - j3$, $d_3 = 1 + j5$, $d_4 = 1 - j5$.

Hence the roots will be

$(- a) = X - d_1 = - (2 + j3)$, $(- b) = X - d_2 = - (2 - j3)$,

$(- c) = X - d_3 = - (4 + j5)$, $(- e) = X - d_4 = - (4 - j5)$.

(D) Let us take an equation,
$y = (x + a)(x + b)(x + c)(x + e)(x + f) = x^5 + (a + b + c + e + f)x^4 + (ab + ac + ae + af + bc + be + bf + ce + cf + ef)x^3 + (abc + abe + abf + ace + acf + aef + bce + bcf + bef + cef)x^2 + (abce + abcf + abef + acef + bcef)x + abcef = x^5 + Px^4 + Qx^3 + Rx^2 + Sx + T$.

To solve the equation for x when $y = 0$, we adopt the following steps: (1) Find $\frac{d^4y}{dx^4} = 0$ and then put $x = X$, an average value of the roots to obtain $X = - \frac{P}{5} = -\frac{a+b+c+e+f}{5}$, or, $P = - 5X$

(2) Put the value of P in the equation and find the value of $y = Y = x^5 + Px^4 + Qx^3 + Rx^2 + Sx + T = T + SX + RX^2 + QX^3 - 4X^5$.

If $Y = 0$, then X is one root of the equation.

(3) Find the values of the roots $(- a) = X - d_1$, $(- b) = X - d_2$, $(- c) = X - d_3$, $(- e) = X - d_4$ and $(- f) = X - d_5$, where d_1, d_2, d_3, d_4, d_5 are termed as deviations from the average value of the roots. To find (- a), (- b), (- c), (- e) and (- f), put

(i) $d_1 + d_2 + d_3 + d_4 + d_5 = 0$,

(ii) $d_1 d_2 + d_1 d_3 + d_1 d_4 + d_1 d_5 + d_2 d_3 + d_2 d_4 + d_2 d_5 + d_3 d_4 + d_3 d_5 + d_4 d_5 = Q - 10X^2$,

(iii) $d_1d_2d_3 + d_1d_2d_4 + d_1d_2d_5 + d_1d_3d_4 + d_1d_3d_5 + d_1d_4d_5 +$
$d_2d_3d_4 + d_2d_3d_5 + d_2d_4d_5 + d_3d_4d_5 = R + 3QX - 20X^3$,

(iv) $d_1d_2d_3d_4 + d_1d_2d_3d_5 + d_1d_2d_4d_5 + d_1d_3d_4d_5 + d_2d_3d_4d_5 =$
$S + 2RX + 3QX^2 - 15X^4$,

(v) $d_1d_2d_3d_4d_5 = Y = T + SX + RX^2 + QX^3 - 4X^5$

[Hints: To prove (3)(i),(ii),(iii),(iv),(v) we take

$-(a + b + c + e + f) = - P = (5X - d_1 - d_2 - d_3 - d_4 - d_5)$,

$(ab + ac + ae + af + bc + be + bf + ce + cf + ef) = Q =$
$(X - d_1)(X - d_2) + (X - d_1)(X - d_3) + (X - d_1)(X - d_4) +$
$(X - d_1)(X - d_5) + (X - d_2)(X - d_3) + (X - d_2)(X - d_4) +$
$(X - d_2)(X - d_5) + (X - d_3)(X - d_4) + (X - d_3)(X - d_5) +$
$(X - d_4)(X - d_5)$,

$- (abc + abe + abf + ace + acf + aef + bce + bcf + bef + cef) =$
$- R = (X - d_1)(X - d_2)(X - d_3) + (X - d_1)(X - d_2)(X - d_4) +$
$(X - d_1)(X - d_2)(X - d_5) + (X - d_1)(X - d_3)(X - d_4) +$
$(X - d_1)(X - d_3)(X - d_5) + (X - d_1)(X - d_4)(X - d_5) +$
$(X - d_2)(X - d_3)(X - d_4) + (X - d_2)(X - d_3)(X - d_5) +$
$(X - d_2)(X - d_4)(X - d_5) + (X - d_3)(X - d_4)(X - d_5)$,

$(abce + abcf + abef + acef + bcef) = S =$
$(X - d_1)(X - d_2)(X - d_3)(X - d_4) +$
$(X - d_1)(X - d_2)(X - d_3)(X - d_5) +$
$(X - d_1)(X - d_2)(X - d_4)(X - d_5) +$
$(X - d_1)(X - d_3)(X - d_4)(X - d_5) +$
$(X - d_2)(X - d_3)(X - d_4)(X - d_5)$ and

$- abcef = -T = (X - d_1)(X - d_2)(X - d_3)(X - d_4)(X - d_5)$.]

(4) To solve for d_1, d_2, d_3, d_4, d_5, let $d_1 = A$, $d_2 = B + jC$
$d_3 = B - jC$, $d_4 = D + jE$, $d_5 = D - jE$. Then we get from (3)(i),

$$d_1 + d_2 + d_3 + d_4 + d_5 = A + 2B + 2D = 0.$$

Again we get from (3)(ii), $(B^2 + C^2) + (D^2 + E^2) + 4BD - A^2 =$
$(Q - 10X^2)$.

Again we get from (3)(iii), $-2B(B^2 + C^2) - 2D(D^2 + E^2) + 4ABD = (R + 3QX - 20X^3)$.
Again we get from (3)(iv), $2AD(B^2 + C^2) + 2AB(D^2 + E^2) + (B^2 + C^2)(D^2 + E^2) = (S + 2RX + 3QX^2 - 15X^4)$ and from (3)(v),

$A(B^2 + C^2)(D^2 + E^2) = Y = (T + SX + RX^2 + QX^3 - 4X^5)$.

Then we get, $A^5 + (Q - 10X^2)A^3 - (R + 3QX - 20X^3)A^2 + (S + 2RX + 3QX^2 - 15X^4)A - (T + SX + RX^2 + QX^3 - 4X^5) = 0$.
This is the form as following: $A^5 + MA^3 + NA^2 + KA + L = 0$.
We may solve this equation by the following ways:
(a) We convert the equation such that,

$$A^5 + MA^3 + NA^2 + KA + L = (A^3 + pA + q)(A^2 + r) = 0.$$

Then we get, $p = \frac{KN}{L}$, $q = N$, $r = \frac{L}{N}$ when $M = \frac{KN}{L} + \frac{L}{N}$.
Hence, $(A^3 + pA + q) = 0$, or, $(A^2 + r) = 0$.
(b) If the value of M is not equal to $(\frac{KN}{L} + \frac{L}{N})$, then the above equation can not be solved by the above way (a). Then we factorize the value of L such that $mn = L$, or, $n = \frac{L}{m}$.
Then we use the values of m and n to get,

$$m^4 + Mm^2 + Nm + n = -K$$

Hence $A = m$ or $-m$ will be one real root of the above equation.
Then we can find the values of B, C, D and E. Hence $(-a)$, $(-b)$, $(-c)$, $(-e)$, $(-f)$ can be found out.
Examples:- (i) Let, $y = x^5 + 20x^4 + 155x^3 + 580x^2 + 1044x + 720 = x^5 + Px^4 + Qx^3 + Rx^2 + Sx + T$.

Then, $\frac{d^4y}{dx^4} = 120x + 480$. So, $120X + 480 = 0$, or, $X = -4$.
Then, $(Q - 10X^2) = 155 - 10(4)^2 = -5$, $(R + 3QX - 20X^3) = (580) - 3(155)(4) + 20(4)^3 = 0$, $(S + 2RX + 3QX^2 - 15X^4) = 1044 - 2(580)(4) + 3(155)(4)^2 - 15(4)^4 = 4$, $(T + SX +$

$RX^2 + QX^3 - 4X^5) = 720 - 1044(4) + 580(4)^2 - 155(4)^3 + 4(4)^5 = 0$. Hence, $A^4 - 5A^2 + 4 = 0$, or, $(A^2 - 1)(A^2 - 4) = 0$, or, $A = \pm 1, \pm 2$. Then, $d_1 = \pm 1, \pm 2$, $d_2 = 0, \pm 1$, $d_3 = 0, \pm 1$, $d_4 = \pm 1, \pm 2$, $d_5 = \pm 1, \pm 2$. Hence the roots will be $(- a) = X - d_1 = - 2$, $(- b) = X - d_2 = - 3$, $(- c) = X - d_3 = - 4$, $(- e) = X - d_4 = - 5$, $(- f) = X - d_5 = - 6$.

(ii) Let, $y = x^5 + 15x^4 + 112x^3 + 488x^2 + 1215x + 1625 = x^5 + Px^4 + Qx^3 + Rx^2 + Sx + T$.

Then, $\dfrac{d^4y}{dx^4} = 120x + 360$. So, $120X + 360 = 0$, or, $X = - 3$. So we get, $A^5 + MA^3 + NA^2 + KA + L = A^5 + 22A^3 - 20A^2 + 96A - 320 = 0$. Hence we get,

$$M = \frac{KN}{L} + \frac{L}{N}, \quad \text{or,} \quad 22 = \frac{(96)(20)}{320} + \frac{320}{20} = 22$$

So we convert the equation such that,

$$A^5 + 22A^3 - 20A^2 + 96A - 320 = (A^3 + 6A - 20)(A^2 + 16) = 0.$$

Then, $(A^3 + 6A - 20) = 0$, or, $(A^2 + 16) = 0$. So one real value will be $A = d_1 = 2$. Hence the roots will be,

$(- a) = - 5$, $(- b) = - (2 + j3)$, $(- c) = - (2 - j3)$, $(- e) = - (3 + j4)$, $(- f) = - (3 - j4)$.

(iii) Let, $y = x^5 + 30x^4 + 397x^3 + 2768x^2 + 10500x + 20000 = x^5 + Px^4 + Qx^3 + Rx^2 + Sx + T$.

Then we get, $A^5 + MA^3 + NA^2 + KA + L = A^5 + 37A^3 + 58A^2 + 720A - 2000 = 0$. But, $M \neq \dfrac{KN}{L} + \dfrac{L}{N}, or, 37 \neq -\dfrac{(720)(58)}{2000} - \dfrac{2000}{58}$

Now we factorize the value of L such that, $m = 2$, $n = - 1000$, or, $mn = 2x(- 1000)$. Then we get, $m^4 + Mm^2 + Nm + n = - K$, or, $16 + 148 + 116 - 1000 = - 720$. Hence,

$A = m = 2$, or, $d_1 = 2$. So, $(- a) = - 8$, $(- b) = - (3 + j4)$, $(- c) = - (3 - j4)$, $(- e) = - (8 + j6)$, $(- f) = - (8 - j6)$.

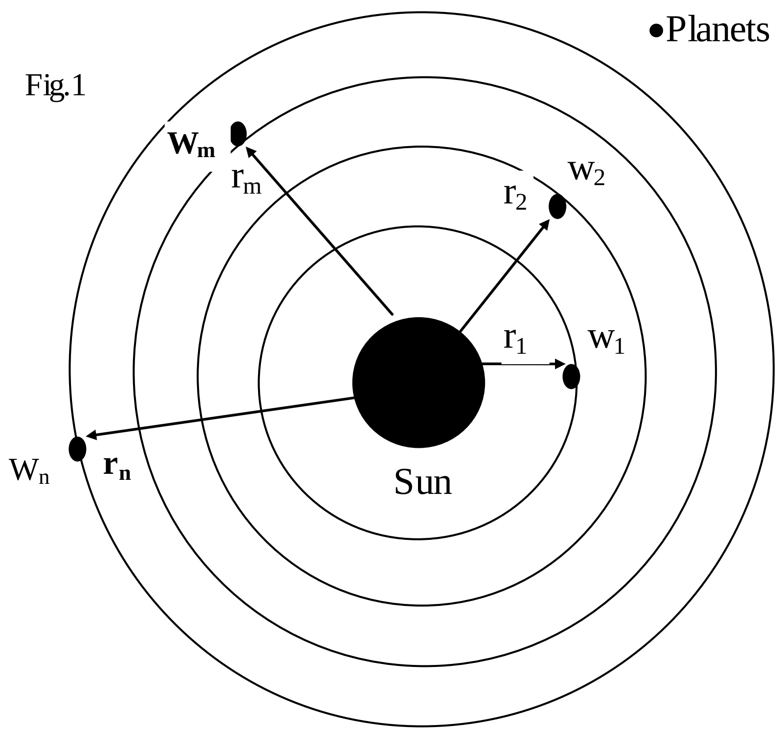
PART-VII

AGE OF A LIVING BODY ON OTHER PLANETS & RELATIVITY

Introduction: The famous scientist Einstein showed that for synchronizing separated stationary clocks, two observers in relative motion disagree in their measurements of time and distance intervals. The planets in this universe rotate around the sun with different velocities and different distances. On the basis of above, mathematical calculations are shown here to indicate the age of a living body on other planets.

Derivation: Let us consider the following Fig.1 showing the different angular velocities of the planets at different distances from the sun. Let w_m , w_n are the angular velocities and v_m , v_n

Fig.1



are the linear velocities of the m_{th}, n_{th} planets and r_m, r_n are the distances of the planets from the sun. Let T_m, T_n are the time required for one complete rotation of the m_{th}, n_{th} planets around the sun and A_m, A_n are the ages of a living body on the respective planets (if it is possible to transfer a living body from one planet to another).Then we get, $v_m = w_m r_m$ and $v_n = w_n r_n$.

$$\text{So, } \frac{v_m}{v_n} = \frac{r_m T_n}{r_n T_m}.$$

Let the age of a living body on a planet is the time of total rotations around the sun. Then we get,

$$A_m = C_m T_m \quad \text{and} \quad A_n = C_n T_n,$$

where C_m, C_n are the number of rotations and dimensionless units. Then,

$$\frac{A_m}{A_n} = \frac{C_m T_m}{C_n T_n} = \frac{C_m r_m v_n}{C_n r_n v_m}$$

The relativity indicates that time will increase at the higher n_{th} planet for the same number of rotations of the planets, that is for

$C_m = C_n$. So we get, $\dfrac{A_m}{A_n} = \dfrac{r_m v_n}{r_n v_m} = \dfrac{w_n}{w_m}$.

Hence it is understood that the age of a living body on a planet is directly proportional to the distance of the planet from the sun if the velocities $v_1, v_2, \ldots\ldots v_m, v_n$ of the planets are same. It is also understood that the age is inversely proportional to the angular velocity of a planet with respect to sun.

These results are clearly stated in Srimad-Bhagavatam: 4/25/43.

Reference: Relativity - International Science & Technology, March 1966, No.51-written by Gerard Fournet and Robert Bonnefile.

LAST THEOREM OF P. FERMAT

(with other topics)

Prof. Ramesh Chandra Ray
Electrical & Electronic Engineering
CUET, Chittagong, Bangladesh
E-mail:ramesh_ray2003@yahoo.com
E-mail:rameshchandraray@yahoo.com

Photo

Critics Praise

There is little question that this edition is one of the authoritative books available on mathematics. The author does a real service to the students by discussing simple analysis on P. Fermat's last theorem and some other topics.

Prof. Dr. Amal Krishna Halder
Address: Department of Mathematics
Dhaka University, Dhaka, Bangladesh

It is a deeply felt, powerfully conceived and beautifully explained work on mathematics. It will occupy a significant place in the intellectual life of the modern mathematicians for a long time to come.

J. B. Prasad
Editor, THE MATHEMATICS EDUCATION
SIWAN, Bihar, India

Printed at: Kakali Press
Ahsan Ahmed Road
Khulna, Bangladesh

Price: Taka-20/- (Twenty) only