

Due Date: *The assignment should be completely solved and be ready to be turned in all lecture hours 1 week after the post date.*

Not:

1. Put your name on the **back** of the assignment only.
2. Answer the questions on standard A4 sheets.
3. Use both side of each sheet.
4. Leave at least 1 inch (2.5cm) margin on each side of the page.
5. Staple all the pages together **at the upper left corner**.
6. You must **show your work** to receive full credit.
7. Good Luck.

1-) Verify the equation 2.3.5 by using the properties of expected value.

2-) X is a discrete random variable that indicates the number when a fair dice thrown once.

- a) Indicate the probability density function with graph.
- b) What is the probability of $X=4$? What is the probability of $X=4$ or $X=5$?
- c) What is the expected value of X ? In this case, explain the meaning of the expected value of X .
- d) Find the expected value of X^2 .
- e) Find the variance of X .

3-) Let X be a continuous random variable with a probability density function is given below.

$$f(x) = \begin{cases} 2x & 0 \leq x \leq 1 \\ 0 & x < 0 \text{ ve } x > 1 \end{cases}$$

- a) Draw the graph of $f(x)$.
- b) Calculate the probability that X is between 0 and $\frac{1}{2}$ and verify the result geometrically.
- c) Calculate the probability that X is between $\frac{1}{4}$ and $\frac{3}{4}$ and verify the result geometrically.

4-) Let X_1 and X_2 are both independent random variables with probability density function $f(x)$ in Exercise 3.

What is the joint probability density function $f(x_1, x_2)$ of X_1 and X_2 ? Find.

5-) The joint probability density function of the random variables Gender (C) and Political Tendency (S) is as below.

		C		
S		0	1	$h(s)$
	0	0.20	0.27	0.47
	1	0.30	0.10	0.40
	2	0.06	0.07	0.13
	$f(c)$	0.56	0.44	1.00

$$S = \begin{cases} 0 & \text{Democrat} \\ 1 & \text{Conservative} \\ 2 & \text{Other} \end{cases} \quad C = \begin{cases} 0 & \text{Male} \\ 1 & \text{Female} \end{cases}$$

- a) Are the the random variables Gender (C) and Political Tendency (S) independent? If not, why?
b) Find the conditional probability distribution $f(s|c=0)$.

6-) The probability density function of discrete random variable X is $f(x)=p^x(1-p)^{1-x}$ for $x=0,1$.

- a) Find the mean and variance of X.
b) $X_1, X_2, \dots, X_n$ are independent random variables with (0,1) and each has probability density function $f(x)$.
The random variable $B= X_1+ X_2+ \dots +X_n$ has binomial distribution with parameters n and p . The values of B are $b = 0,1,2,\dots,n$ that represents the “success” with probability p in n independent experiment ($X_i = 1$).
Calculate the mean and variance of B .
c) The random variable $Y=B/n$ refers to the success rate in n experiment. Find the mean and variance of Y .

7-) Show geometrically that mean of a random variable that has uniform distribution is $(a+b)/2$.

8-) The joint probability density function of discrete random variables X and Y is as follows:

		Y		
		1	3	9
X	2	1/8	1/24	1/12
	4	1/4	1/4	0
	6	1/8	1/24	1/12

- a) Find the marginal density function for Y.
b) Find the conditional probability density function of Y for $X = 2$.
c) Find the covariance of X and Y.
d) Are the X and Y independent?
- 9-) A fair dice is thrown twice. Let X_1 and X_2 represent the results in the first and second throw respectively. .
 $U=X_1+X_2$ and $V=X_1-X_2$.
a) Find the mean and variance of V.
b) Find the covariance of U and V.
c) Prove that U and V are not independent.
- 10-) X_1 and X_2 are independent discrete random variables. Show that $E[X_1X_2] = E[X_1].E[X_2]$ where joint probability density function is $f(x_1,x_2)$.