Fall 2018 **Linear Algebra Review HW** **Kenan Lopcu**

Due Date: *The assignment should be completely solved and be ready to be turned in all lecture hours 1 week after the post date.*

Instructions:

1. Put your name on the **back** of the assignment only.
2. Answer the questions on standard A4 sheets.
3. Use both side of each sheet.
4. Leave at least 1 inch (2.5cm) margin on each side of the page.
5. Staple all the pages together **at the upper left corner**.
6. You must **show your work** to receive full credit.
7. Good luck.

1. Solve the following system by the method of reduction.

$$\begin{aligned} x_1 + 3x_2 - 2x_3 \quad + 2x_5 \quad &= 0 \\ 2x_1 + 6x_2 - 5x_3 - 2x_4 + 4x_5 - 3x_6 &= -1 \\ 5x_3 - 10x_4 \quad + 15x_6 &= 5 \\ 2x_1 + 6x_2 \quad + 8x_4 + 4x_5 + 18x_6 &= 6 \end{aligned}$$

2. For which values of *a* will the following system have no solutions? Exactly one solution? Infinitely many solutions?

$$\begin{aligned} x + 2y \quad - 3z &= 4 \\ 3x - y \quad + 5z &= 2 \\ 4x + y + (a^2 - 14)z &= a + 2 \end{aligned}$$

3. Without using pencil and paper, determine which of the following homogeneous systems have nontrivial solutions.

(a) $$\begin{aligned} x_1 + 3x_2 + 5x_3 + x_4 &= 0 \\ 4x_1 - 7x_2 - 3x_3 - x_4 &= 0 \\ 3x_1 + 2x_2 + 7x_3 + 8x_4 &= 0 \end{aligned}$$

(b) $$\begin{aligned} x_1 + 2x_2 + 3x_3 &= 0 \\ x_2 + 4x_3 &= 0 \\ 5x_3 &= 0 \end{aligned}$$

(c) $$\begin{aligned} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 &= 0 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 &= 0 \end{aligned}$$

(d) $$\begin{aligned} x_1 + x_2 &= 0 \\ 2x_1 + 2x_2 &= 0 \end{aligned}$$

4.Consider the following system: (a) Solve the system by Cramer's rule (b) Solve the system by the method of reduction (c) Find the inverse of the coefficients matrix, A^{-1}, using $A^{-1} = \frac{(A^*)^T}{DET(A)}$ (d) Find the inverse of the coefficients matrix, A^{-1} by applying elementary row operations to $[A|I]$ and obtaining $[I|A^{-1}]$ (e) Solve the system by using A^{-1}.

$$\begin{aligned} 4x + y + z + w &= 6 \\ 3x + 7y - z + w &= 1 \\ 7x + 3y - 5z + 8w &= -3 \\ x + y + z + 2w &= 3 \end{aligned}$$

5. Condider the vectors: **u**=(2,0,-1,3), **v**=(5,4,7,-1), **w**=(6,2,0,9). Find the vector **x** that satisfies **2u-v+x=7x+w**.

6. Let $\mathbf{u_1}$=(-1,3,2,0), $\mathbf{u_2}$=(2,0,4,-1), $\mathbf{u_3}$=(7,1,1,4), $\mathbf{u_4}$=(6,3,1,2). Find scalars c_1,c_2,c_3, and c_4 such that $c_1\mathbf{u_1}+c_2\mathbf{u_2}+c_3\mathbf{u_3}+c_4\mathbf{u_4}$=(0,5,6,-3).

7. Which of the following are linear combinations of **u**=(1,-1,3) and **v**=(2,4,0)?

(a) (3,3,3) (b) (4,2,6) (c) (1,5,6) (d) (0,0,0)

8. Which of the following are linear combinations of

$$A=\begin{bmatrix}1 & 2\\ -1 & 3\end{bmatrix} \quad B=\begin{bmatrix}0 & 1\\ 2 & 4\end{bmatrix} \quad \text{and } C=\begin{bmatrix}4 & -2\\ 0 & -2\end{bmatrix}?$$

(a) $\begin{bmatrix}6 & 3\\ 0 & 8\end{bmatrix}$ (b) $\begin{bmatrix}-1 & 7\\ 5 & 1\end{bmatrix}$ (c) $\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$ (d) $\begin{bmatrix}6 & -1\\ -8 & -8\end{bmatrix}$

9. Let $A=I-X(X^TX)^{-1}X^T$

(a) Must A be square (square matrix)? (b) Must (X^TX) be square?

(c) Must X be square? (d) Is A idempotent?

10. Show that the Cramer's rule based upon the concept of the inverse matrix (although in practice it bypasses the process of matrix inversion).

Hint: First, note that given a system $AX=b$, the solution is $X=A^{-1}.b = \frac{1}{DET(A)} adj(A).b$, and then substitute *adj(A)*.

11. Suppose there are only three industries, x_1, x_2, and x_3 in the economy. Input coefficient matrix and final demand vector are as follows:

$$A=\begin{bmatrix}a_{11} & a_{12} & a_{13}\\ a_{21} & a_{22} & a_{23}\\ a_{31} & a_{32} & a_{33}\end{bmatrix}=\begin{bmatrix}0.2 & 0.3 & 0.2\\ 0.4 & 0.1 & 0.2\\ 0.1 & 0.3 & 0.2\end{bmatrix} \qquad d=\begin{pmatrix}d_1\\ d_2\\ d_3\end{pmatrix}=\begin{pmatrix}10\\ 5\\ 6\end{pmatrix}$$

(a) Find the amount of primary input (input not produced by an industry) for each unit of output for each industry.

(b) Find the production level for each industry.

(c) Find the required primary input needed to support the total production level of the economy.

(d) Find the production level for each industry if d_1 increases to 20.

(e) Find the required primary input needed to support the production level in the economy after the increase in d_1.

12. Consider the coefficients matrix, A, in problem 11.

(a) Calculate (I-A), A^2,A^3, A^4, and A^5.

(b) Calculate (I-A)(I+A), (I-A)($I+A+A^2$), (I-A)($I+A+A^2+A^3$), and (I-A)($I+A+A^2+A^3+A^4$).

(c) What can you conclude about the matrix ($I+A+A^2+\ldots A^{p-1}+A^p$) as p increases. Prove your claim.