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Algebraic solution of Fermat's theorem
(mathematics, number theory, updated version)

Abstract: *Fermat's Last Theorem (or Fermat's last theorem) is one of the most popular theorems in mathematics. Formulated in French mathematician Pierre Fermat in 1637. Despite the simplicity of the formulation, literally, at the “school” arithmetic level, proof of the theorem sought by many mathematicians for more than three hundred years. And only in 1994 year the theorem was proven by the English mathematician Andrew Wilson with colleagues; The proof was published in 1995. [1]-[5]*

With this article, the author completes his research on the given topic, makes corrections and eliminates the errors of the previous ones.

Keywords: *Theorem, Fermat, elementary, solution*

Introduction.

$$X^n + Y^n = Z^n \quad (01)$$

where:

n- prime number; $n > 2$; X, Y, Z are integers.

The solutions of which can be X, Y, Z - relatively prime numbers.

The author's solution is based on the following:

For a polynomial of integers containing a common prime factor n present in each monomial of varying degrees m , a necessary condition for the solution in integers is to find this factor with the smallest degree in more than one monomial.

1. Decomposition of (01) into multipliers.

If n is odd, then (01) will decompose into multipliers:

$$X^n + Y^n = (X + Y)(X^{n-1} - X^{n-2}Y + \dots - XY^{n-2} + Y^{n-1}) \quad (02)$$

where in the second bracket is the geometric progression

first term $a_1 = X^{n-1}$, and a multiplier $q = -\frac{Y}{X}$

The sum of the members of which $S = \frac{a_1(1 - q^n)}{1 - q}$

$$Z^n = Z_{11} Z_{22} \quad (03)$$

where:

$$Z_{11} = X + Y \quad (04)$$

$$Z_{22} = X^{n-1} - X^{n-2}Y + \dots - X Y^{n-2} + Y^{n-1} \quad (05)$$

2. Equivalent representation Z_{22} .

If we sum the equidistant terms from the middle term of the progression Z_{22} in pairs.

of the middle term of the progression in pairs we have:

for degree 3

$$Z_{22} = (X + Y)^2 - 3XY \quad (06)$$

Fifth degree :

$$Z_{22} = \frac{X^5 + Y^5}{X + Y} = X^4 - X^3 Y + X^2 Y^2 - X Y^3 + Y^4 \quad (07)$$

$$X^4 + Y^4 = (X + Y)^4 - 4 X Y (X + Y)^2 + 2 X^2 Y^2 \quad (08)$$

$$-X Y^3 - X^3 Y = -X Y (X^2 + Y^2) = -X Y (X + Y)^2 + 2 X^2 Y^2 \quad (09)$$

$$Z_{225} = (X + Y)^4 - 5(X + Y)^2 + 5 X^2 Y^2 \quad (10)$$

to the 7th degree:

$$Z_{227} = (X + Y)^6 - 7 X Y (X + Y)^4 + 14 X^2 Y^2 (X + Y)^2 - 7 X^3 Y^3 \quad (11)$$

degree n :

$$\begin{aligned} Z_{22N} &= \frac{X^n + Y^n}{X + Y} = \\ &= (X + Y)^{n-1} - K_{n-3} X Y (X + Y)^{n-3} + \dots \mp K_2 X^{\frac{n-3}{2}} Y^{\frac{n-3}{2}} (X + Y)^2 \pm n X^{\frac{n-1}{2}} Y^{\frac{n-1}{2}} \end{aligned} \quad (12)$$

$$Z_{22N} = (X + Y)^{n-1} - K_{n-3} X Y (X + Y)^{n-3} + \dots \mp K_2 X^{\frac{n-3}{2}} Y^{\frac{n-3}{2}} (X + Y)^2 \pm n X^{\frac{n-1}{2}} Y^{\frac{n-1}{2}} \quad (13)$$

where $K_{n-3} \dots K_2$ corresponding coefficients at $(XY) \dots (X+Y) \dots$

equivalent representation Z_{22N} algebraic sum of even powers of

$X+Y$ and the residual term $\pm n X^{\frac{n-1}{2}} Y^{\frac{n-1}{2}}$.

Note that by n we mean in Mathematical deduction1 any odd power, in Mathematical deduction2 the power of an odd prime number.

Mathematical deduction1.

Suppose that for an n odd number and for the previous $n-2$

an equivalent representation(13) is valid, then for the next $n+2$ it is

(13) is valid.

We show the transition from the two previous odd degrees to the next one further:

$$Z_n^n = X^n + Y^n \quad (14)$$

$$(X^{n-2} + Y^{n-2})(X^2 + Y^2) = X^n + Y^n + Y^2 X^{n-2} + Y^2 Y^{n-2} \quad (15)$$

$$Z_n^n = X^n + Y^n = (X^{n-2} + Y^{n-2})(X^2 + Y^2) - X^2 Y^2 (X^{n-4} + Y^{n-4}) \quad (16)$$

details:

$$\frac{X^{n_1} + Y^{n_1}}{X + Y} \text{ multiply by } (X + Y)^2 - 2XY$$

$$X^{n+2} + Y^{n+2} = (X + Y)[Z_{22N}(X + Y)^2 - 2XYZ_{22N} - X^2 Y^2 (X^{n-2} + Y^{n-2})] \quad (17)$$

$$(X + Y)^{n-1} - K_{n-3}XY(X + Y)^{n-3} + \dots \mp K_2 X^{\frac{n-3}{2}} Y^{\frac{n-3}{2}} (X + Y)^2 \pm nX^{\frac{n-1}{2}} Y^{\frac{n-1}{2}} *$$

$$* (X + Y)^2 = (X + Y)^{n+1} - K_{n-1(01)}XY(X + Y)^{n-1} + \dots \mp K_{4(01)}X^{\frac{n-3}{2}} Y^{\frac{n-3}{2}} \pm nX^{\frac{n-1}{2}} Y^{\frac{n-1}{2}} (X + Y)^2$$

(18)

$$-2XY \left[(X + Y)^{n-1} - K_{n-3}XY(X + Y)^{n-3} + \dots \mp K_2 X^{\frac{n-3}{2}} Y^{\frac{n-3}{2}} (X + Y)^2 \pm nX^{\frac{n-1}{2}} Y^{\frac{n-1}{2}} \right] =$$

=

$$-2XY(X + Y)^{n-1} - K_{n-3(02)}2X^2 Y^2 (X + Y)^{n-3} + \dots \mp K_{2(02)}2X^{\frac{n-1}{2}} Y^{\frac{n-1}{2}} (X + Y)^2 \pm 2X^{\frac{n+1}{2}} Y^{\frac{n+1}{2}} n$$

(19)

$$-X^2 Y^2 (X + Y)^{n-3} + K_{n-3(03)}X^3 Y^3 (X + Y)^{n-5} + \dots \pm K_{2(03)}X^{\frac{n-1}{2}} Y^{\frac{n-1}{2}} (X + Y)^2 \mp (n-2)X^{\frac{n+1}{2}} Y^{\frac{n+1}{2}}$$

(20)

where $K_{...(01)}$ - corresponding coefficients when multiplied by $(X + Y)^2$,

$K_{...(02)}$ - corresponding coefficients when multiplied by $-2XY$,

$K_{...(03)}$ - corresponding coefficients when multiplied by $-X^2 Y^2$

After adding these algebraic terms we again obtain(13)

Mathematical deduction2.

The equivalent representation (13) is valid for any prime n.

By Mathematical deduction 1 , if the two previous representations of (13) are valid, of degree 3 and 5, then it is valid for degree 7. Now

taking the previous 5 and 7 degrees we have its validity for the 9th degree, etc, which means all odd degrees are described by the above formula.

And since it includes odd prime , it is valid for prime n .

Let us represent (01) as according to Newton's binomial:

$$\begin{aligned} (X+Y)^n - Z^n &= nX^{n-1}Y + \frac{n(n-1)}{2}X^{n-2}Y^2 + \dots + \frac{n(n-1)}{2}Y^{n-2}X^2 + nXY^{n-1} \\ &= [(X+Y)-Z]^n - n(X+Y)Z * \dots \end{aligned} \quad (21)$$

From (21) it follows that $X+Y - Z$ is divisible by n and further:

$$\begin{aligned} (X+Y-Z)[(X+Y-Z)^{n-1} - nk_{n-3}(X+Y)Z(X+Y-Z)^{n-3} \pm nk_2(X+Y)^{n-3}Z^{n-3}(X+Y-Z)^2 \mp n(XY)^{\frac{n-1}{2}}] \\ = nX^{n-1}Y + \frac{n(n-1)}{2}X^{n-2}Y^2 + \dots + \frac{n(n-1)}{2}Y^{n-2}X^2 + nXY^{n-1} \end{aligned} \quad (22)$$

it follows:

$$(X+Y)[(X+Y)^{n-1} - \dots \mp n(XY)^{\frac{n-1}{2}}] = Z^n \quad (23)$$

$$Z_{11n} = X+Y \quad (24)$$

$$Z_{22n} = (X+Y)^{n-1} - nk_{n-3}XY(X+Y)^{n-3} + \dots \pm nk_2X^{\frac{n-3}{2}}Y^{\frac{n-3}{2}}(X+Y)^2 \mp nX^{\frac{n-1}{2}}Y^{\frac{n-1}{2}} \quad (25)$$

3. Analysis of Equation (25).

a) From equation (25) $Z_{11} = X+Y$ and Z_{22} cannot have a common factor for except for n . Z_{22} consists of members each of which has a factor $X+Y$, with the exception of the last product $nX^{\frac{n-1}{2}}Y^{\frac{n-1}{2}}$, which in the case of a common factor c $X+Y$ must involve factors of either X and Y , and they are coprime, so (26). From which the following equalities follow in the absence of n :

$$X+Y=Z_1^n, \quad Z-X=Y_1^n, \quad Z-Y=X_1^n \quad (26)$$

$$Z_{11}=Z_1^n, \quad Z_{22}=Z_2^n, \quad X_{11}=X_1^n, \quad X_{22}=X_2^n, \quad Y_{11}=Y_1^n, \quad Y_{22}=Y_2^n \quad (27)$$

$$X+Y-Z=n X_1 Y_1 Z_1 K_o \quad (28)$$

where

K_o -an integer coprime to the others specified

except n .

$$Z_1^n = X_1^n + Y_1^n + 2 n X_1 Y_1 Z_1 K_o \quad (29)$$

$$X - Y = X_1^n - Y_1^n \quad (30)$$

$$Z_1^n - Z = n X_1 Y_1 Z_1 K_o \quad (31)$$

$$Z_2 = Z_1^{n-1} - n X_1 Y_1 K_o \quad (32)$$

$$X - X_1^n = n X_1 Y_1 Z_1 K_o \quad (33)$$

$$X_2 = X_1^{n-1} + n Z_1 Y_1 K_o \quad (34)$$

$$Y - Y_1^n = n X_1 Y_1 Z_1 K_o \quad (35)$$

$$Y_2 = Y_1^{n-1} + n Z_1 X_1 K_o \quad (36)$$

$$2 X = Z_1^n - Y_1^n + X_1^n \quad (37)$$

$$2 Y = Z_1^n - X_1^n + Y_1^n \quad (38)$$

$$2 Z = Z_1^n + X_1^n + Y_1^n \quad (39)$$

$$Z_1^n - X_1^n - Y_1^n = 2 n X_1 Y_1 Z_1 K_o \quad (40)$$

$$Z_1^n - [(X_1 + Y_1)^n - n X_1^{n-1} Y_1 - \dots - n Y_1^{n-1} X_1] = 2 n X_1 Y_1 Z_1 K_o \quad (41)$$

из чего вытекает:

$$Z_1 - X_1 - Y_1 = n K_n \text{ from which it follows } Z_1 > n \quad (42)$$

Note X, Y, Z are coprime numbers, as well as $X_1, X_2, Y_1, Y_2, Z_1, Z_2$.

If the sum or difference of two coprime numbers has a factor n , then

the sum and difference of the n -power of these numbers is divisible by at least n^2 , which is obvious from (25), (04).

b) If in the expansion Z, X, Y has a prime factor n

$$Z_{22} = nZ_2^n, \quad X_{22} = nX_2^n, \quad Y_{22} = nY_2^n \quad (43)$$

and according to formula (25) Z_2 cannot have n available, otherwise

this will lead to the presence of it in X or Y , and vice versa, which is not acceptable.

Z_2, X_2, Y_2 - does not contain the factor n . In this regard, if Z contains a factor n , then

formula (26) has the form, since sum $X_1^n + Y_1^n$ contains a multiplier n^m where natural number, $m \geq 2$

$$n^{nm-1} Z_1^n = X_1^n + Y_1^n + 2n^m X_1 Y_1 Z_1 K_o \quad (44)$$

To solve (39) in integers, degree n in $X_1^n + Y_1^n$, should be equal

degree n in the last monomial, that is, minimally n^2 .

similar:

$$n^{nm-1} X_1^n = Z_1^n - Y_1^n - 2n^m X_1 Y_1 Z_1 K \quad (45)$$

$$n^{nm-1} Y_1^n = Z_1^n - X_1^n - 2n^m X_1 Y_1 Z_1 K \quad (46)$$

$$n^{nm-1} Z_1^n - n^m Z_{2n} Z_1 = n^m X_1 Y_1 Z_1 K \quad (47)$$

$$n^m X_{2n} X_1 - n^{nm-1} X_1^n = n^m X_1 Y_1 Z_1 K \quad (48)$$

$$n^m Y_{2n} Y_1 - n^{nm-1} Y_1^n = n^m X_1 Y_1 Z_1 K \quad (49)$$

where:

$$Z_2 = n^m Z_{2n}, Z = Z_1 Z_2 \quad (50)$$

$$X_2 = n^m X_{2n}, X = X_1 X_2 \quad (51)$$

$$Y_2 = n^m Y_{2n}, Y = Y_1 Y_2 \quad (52)$$

Thus

$$X + Y - Z = n X_1 Y_1 Z_1 K_o \text{ universal,}$$

where X_1, Y_1, Z_1, K_o -coprime corresponds to X, Y, Z with and without n . The difference is:

$$K_o = n^{m-1} K \quad (53)$$

Since X, Y, Z are relatively prime numbers, the presence of n in one of them obliges the other two to its absence.

4. Degree $n=3$.

According to (33) and Newton's binomial[6]:

$$\begin{aligned} Z_2^3 &= Z_1^6 - 3(X_1^3 + 3 X_1 Y_1 Z_1 K_o)(Y_1^3 + 3 X_1 Y_1 Z_1 K_o) = (Z_1^2 - 3 X_1 Y_1 K_o)^3 = \\ &= Z_1^6 - 9 Z_1^4 X_1 Y_1 K_o + 27 Z_1^2 X_1^2 Y_1^2 K_o^2 - 27 X_1^3 Y_1^3 K_o^3 \end{aligned} \quad (54)$$

On the other side :

$$\begin{aligned} Z_2^3 &= (X + Y)^3 - 3 X Y = \\ &= Z_1^6 - 3 X_1^3 Y_1^3 - 9 X_1^3 X_1 Y_1 Z_1 K_o - 9 Y_1^3 X_1 Y_1 Z_1 K_o - 27 X_1^2 Y_1^2 Z_1^2 K_o^2 \end{aligned} \quad (55)$$

Underlined in (55) according to (40):

$$-3(X_1^3 + Y_1^3 - Z_1^3 + Z_1^3)3 X_1 Y_1 Z_1 K_o = 2*27 X_1^2 Y_1^2 Z_1^2 K_o^2 - 9 X_1 Y_1 Z_1^4 K_o \quad (56)$$

$$9K_o^3 = 1, \quad K_o^3 = \frac{1}{9} \quad (57)$$

There is no solution in whole numbers.

If Z contains n :

$$X + Y = 3^{3m-1} Z_1^3 \quad (58)$$

$$3 Z_2^3 = 3(3^{3m-1} Z_1^2 - 3^m X_1 Y_1 K_o)^3 = 3^{9m-2} Z_1^6 - 3^{6m+1} Z_1^4 X_1 Y_1 K_o + \\ + 3^{5m+1} Z_1^2 X_1^2 Y_1^2 K_o^2 - 3^{3m+1} X_1^3 Y_1^3 K_o^3 \quad (59)$$

$$(X + Y)^2 - 3 X Y = 3^{6m-2} Z_1^6 - 3 X Y \quad (60)$$

(59)=(60), when divided by 3^2 there is no solution in integers.

And the next solution option according to (25) the difference:

$$(X + Y)^{n-1} - Z_2^n = Z_1^{(n-1)n} - Z_2^n = (Z_1^{n-1} - Z_2^n) * \\ * [(Z_1^{n-1} - Z_2^n)^{n-1} - \dots \pm n Z_1^{(n-1)(\frac{n-1}{2})} Z_2^{\frac{n-1}{2}}] = n^2 * \dots \quad (61)$$

The difference factor (54) n is minimal to the second power. Applicable for $n=3$:

$$Z_1^{2*3} - Z_2^3 = 3 X_1 Y_1 K_o (9 X_1^2 Y_1^2 K_o^2 - 3 X Y) \quad (62)$$

According to (01):

$$Z_2^3 = (X + Y)^2 - 3 X Y \quad (63)$$

Hence $3XY$ is divisible by three squared without remainder, and the need analog X_2 with monomial $3ZY$ or $Y_2 - 3ZX$ requires that two relatively prime numbers have a factor n , which does not exist. There is no solution in integers. When the prime factor 3 is contained in the components X, Y, Z , the solution further (79).

5. Degree n .

It is for this case that we examine the balance of the factor n .

Let us consider equation (01) based on (31), (33), (35):

$$(X_1^n + n X_1 Y_1 Z_1 K_o)^n + (Y_1^n + n X_1 Y_1 Z_1 K_o)^n = (Z_1^n - n X_1 Y_1 Z_1 K_o)^n \quad (64)$$

Let's open the brackets:

$$\begin{aligned} & n \ n \ X_1 \ Y_1 \ Z_1 \ K_o \ X_1^{n(n-1)} + n \ n \ X_1 \ Y_1 \ Z_1 \ K_o \ Y_1^{n(n-1)} + n \ n \ X_1 \ Y_1 \ Z_1 \ K_o \ Z_1^{n(n-1)} + \dots \\ & \dots + 3 * (n \ X_1 \ Y_1 \ Z_1 \ K_o)^n + \underline{X_1^{nn} + Y_1^{nn} - Z_1^{nn}} = 0 \end{aligned} \quad (65)$$

According to (65), the underlined free term is relatively $n \ X_1 \ Y_1 \ Z_1 \ K_o$ is divisible without remainder by n^{m+1} , where m is the degree taking into account its K_o .

Then we have:

$$Z_1^n - X_1^n - Y_1^n \quad (66)$$

is divisible without remainder by n^m and :

$$Z_1^{nn} - X_1^{nn} - Y_1^{nn} \quad (67)$$

is divisible without remainder by n^{m+1} .

What should we proceed from (25):

$$\begin{aligned} Z_1^{nn} - X_1^{nn} - Y_1^{nn} &= Z_1^{nn} - (X_1^{nn} + Y_1^{nn}) = Z_1^{nn} - (X_1^n + Y_1^n)^n + n(X_1^n + Y_1^n)^{n-2} - \dots \pm n(X_1^n Y_1^n)^{\frac{n-1}{2}} * \\ * (X_1^n + Y_1^n) &= \\ &= \underline{Z_1^{nn} - (X_1^n + Y_1^n)^n + n X_1^n Y_1^n (X_1^n + Y_1^n)^{n-2} - \dots \pm n (X_1^n Y_1^n)^{\frac{n-1}{2}} (X_1^n + Y_1^n)} \end{aligned} \quad (68)$$

where the sum of the underlined terms, referring to (25) contains n^{m+1} .

Further, according to (29):

$$Z_1^{n^2-1} - (Z_1^{n-1} - 2 n X_1 Y_1 K_o)(Z_1^n - 2 n X_1 Y_1 Z_1 K_o)^{n-1} + (Z_1^{n-1} - 2 n X_1 Y_1 K_o) * n^{m+1} \dots \quad (69)$$

and then after opening the brackets (other monomials with a factor n^{m+1}):

$$2 n X_1 Y_1 Z_1^{n^2-n} (Z_1^{n-1} - 1) * K_o \quad (70)$$

inevitably divides into a simple n . From which it follows according to (32) $Z_2^n - 1$ also divisible without remainder by prime n .

Similarly:

$$X_1 \frac{[(Z_1^{nn} - Y_1^{nn}) - X_1^{nn}]}{X_1} \quad (71)$$

according to (29):

$$\frac{Z_1^n - Y_1^n}{X_1} = \frac{X_1^n + 2n X_1 Y_1 Z_1 K_o}{X_1} = X_1^{n-1} + 2n Z_1 Y_1 K_o \quad (72)$$

further (25):

$$(X_1^{n-1} + 2n Y_1 Z_1 K_o) (X_1^n + 2n X_1 Y_1 Z_1 K_o)^{n-1} - X_1^{n^2-1} \quad (73)$$

similarly Z_1 :

$$X_1^{n-1} - 1, X_2 - X_1^{n-1}, X_2 - 1, X_2^{n-1} - 1 \text{ - are divisible by } n \quad (74)$$

$$Z^{n-1} - 1, X^{n-1} - 1, Y^{n-1} - 1 \quad (75)$$

is divisible by prime n without remainder:

$$(X + Y)^{n-1} - 1 = (X_1^n + Y_1^n + 2n X_1 Y_1 Z_1 K_o)^{n-1} - 1 = X_1^{(n-1)n} + \dots + Y_1^{(n-1)n} - 1 \quad (76)$$

$$\begin{aligned} \frac{(X + Y)^n}{X + Y} - I &= \frac{(X + Y)^n - (X + Y)}{X + Y} = \frac{(X + Y - Z + Z)^n - (X + Y - Z + Z)}{X + Y} = \\ &= \frac{\dots + Z(Z^{n-1} - 1)}{X + Y} \end{aligned} \quad (77)$$

What follows from this $Z^{n-1} - 1$ is divisible by n without remainder.

Since (74) are divisible by n , we have:

$$X_1^{n-1} - Y_1^{n-1}, Z_1^{n-1} - X_1^{n-1}, Z_1^{n-1} - Y_1^{n-1} \quad (78)$$

$$2 Z_1^{n-1} - X_1^{n-1} - Y_1^{n-1} = n * \dots \quad (79)$$

Note:

The notation $n * \dots$ (or n raised to a power) indicates the absence of the prime factor in the numbers represented by the ellipsis.

which shows the impossibility of solving (01) in integers when one of the components X, Y, Z contains a prime n (leads to its appearance in the others).

According to (31), (33), (35):

$$Z_1^n - Z_1 - Z + Z_1 = n X_1 Y_1 Z_1 K_o \quad (80)$$

From which, obviously, containing one of the three components of a prime n will cause the presence of n in the other two and the solution (01) is absent.

Substitute into (01):

$$(X - X_1 + X_1)^n + (Y - Y_1 + Y_1)^n = (Z - Z_1 + Z_1)^n = \dots + \frac{Z_1^n - X_1^n - Y_1^n}{\dots} \quad (81)$$

marked by the dotted line, is a minimum multiple without remainder n^2 since the remaining terms divide without remainder by n^2 Consider:.

$$Z_1^n - X_1^n - Y_1^n = 2 n^2 X_1 Y_1 Z_1 K \quad (82)$$

From (75) comes:

$$Z_1^n - Z^n = n^2 * \dots \quad (83)$$

And from (70):

$$Z^{n-1} - 1 = n * \dots \quad (84)$$

Multiplying (84) by Z we get:

$$Z^n - Z = n * \dots \quad (85)$$

Let's add (83)+(85):

$$Z_1^n - Z = n^2 X_1 Y_1 Z_1 K \quad (86)$$

In the case of n to the power of $m \geq 2$ (81),(97) the exponent of n is one greater than in the sum of the remaining terms:

$$Z_1^n - Z = n^m X_1 Y_1 Z_1 K \quad (87)$$

and for the solution in integers:

$$Z^n - Z_1^n = n^m * \dots \quad (88)$$

Algebraic sum (87)+(88):

$$Z^n - Z = Z(Z^{n-1} - 1) = n^m * \dots \quad (89)$$

From which it follows:

$$Z - Z_1 = Z_1(Z_2 - 1) = n^{m-1} * \dots \quad (90)$$

In this case, there are monomials in the sum, leading to the appearance of a factor n raised to a power greater than $m-1$. However, the following chain of formulas indicates that the solution in integers is not admissible in this case.

According to (31), (53), the minimum value is (90):

$$Z^{n-1} - 1 = (Z_1^n - n^m X_1 Y_1 Z_1 K)^{n-1} - 1 = n^m * \dots \quad (91)$$

And further :

$$Z_1^{n-1} Z_2^{n-1} - 1 = n^m * \dots \quad (92)$$

According to (70), (32):

$$Z_2 = 1 + n^{m-1} * \dots \quad (93)$$

$$Z_2^{n-1} = (1 + n^{m-1} * \dots)^{n-1} \quad (94)$$

$$Z_1^{n-1} = n^m X_1 Y_1 K + Z_2 = n^m X_1 Y_1 K + 1 + n^{m-1} * \dots \quad (95)$$

In conclusion:

$$Z_2^{n-1} Z_1^{n-1} - 1 = (n^m * \dots + 1 + n^{m-1} * \dots)(1 + n^{m-1} * \dots)^{n-1} - 1 = \dots - n^{m-1} * \dots = n^m * \dots \quad (96)$$

where there is one monomial with a factor n with minimal degree $m-1$ without a pair and therefore (96) is not solvable in integers!

And according to (81):

$$X^n + n(X - X_1) * \dots + X_1^n + Y^n + n(Y - Y_1) * \dots + Y_1^n - Z^n - n(Z - Z_1) * \dots - Z_1^n = 0 \quad (97)$$

Or:

$$n * n^m * \dots - n^m X_1 Y_1 Z_1 K = 0 \quad (98)$$

Confirmation for $m=2$.

Then clearly (75), (84) is divisible without remainder by n^2 and (86) $Z - Z_1$ also contains a multiplier n^2 and what leads to integer division by n^2 (74), (78), (79). We confirm:

$$(Z_1^{n-1} - 1 + 1)(Z_2^{n-1} - 1 + 1) - 1 = (Z_1^{n-1} - 1)(Z_2^{n-1} - 1) + Z_1^{n-1} - 1 + Z_2^{n-1} - 1 = n^2 * \dots \quad (99)$$

The selected number is divided without remainder by n^2 and add and subtract it from (32) taking into account that in (32) on the right side n^2 we have that (74), (78), (79) is divisible

without a remainder on n^2 . (87) shown for n^2 and is applicable for n^m (98).

Let us consider the following according to (25):

$$\begin{aligned} (X + Y)^{n-1} - Z_2^n &= (X + Y - Z + Z)^{n-1} - Z_2^n = \\ &= (X + Y - Z)[(X + Y - Z)^{n-2} + \dots - Z^{n-2}] + Z^{n-1} - 1 - Z_2^n + 1 = Z_1^{(n-1)n} - Z_2^n = n^{m+1} * \dots \\ &\quad \text{----- } n^m \text{ ----- } n^m \text{ ----- } n^{m+1} \end{aligned} \quad (100)$$

From which follows the unique solution for integers, indicated by the line dashes under the formula:

$$n^m * \dots + n^m * \dots - n^{m+1} * \dots = n^{m+1} * \dots \quad (101)$$

In case if in $Z^{n-1} - 1$ does not include the factor n to the power m , then (88) is not solvable in integers relative to prime n .

$Z - Z_1$, $X - X_1$, $Y - Y_1$ thus are divided without remainder into n^m ,

and all other members in (81) marked with ellipses begin with n^{m+1} :

$$n^{m+1} * \dots - Z_1^n - X_1^n - Y_1^n = n^{m+1} * \dots - 2 n^m X_1 Y_1 Z_1 K = 0 \quad (102)$$

(90) has a solution in integers if:

We have an imbalance in n and there are no solutions (102), (81). (01) in integers.

6. Conclusion.

If the degree in (01) is odd, there is no solution. Fermat proved the absence of a solution for the 4th degree and thereby proved its absence for everyone $n=2^m$, where m is an integer. Fermat's theorem is solvable in the first and second powers!

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