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Algebraic solution of Fermat's theorem

(mathematics, number theory)

Abstract: *Fermat's Last Theorem (or Fermat's last theorem) is one of the most popular theorems in mathematics. Formulated in French mathematician Pierre Fermat in 1637. Despite the simplicity of the formulation, literally, at the "school" arithmetic level, proof of the theorem sought by many mathematicians for more than three hundred years. And only in 1994 year the theorem was proven by the English mathematician Andrew Wilson with colleagues; The proof was published in 1995. [1]-[5]*

With this article, the author completes his research on the given topic, makes corrections and eliminates the errors of the previous ones.

Keywords: *Theorem, Fermat, elementary, solution*

Introduction.

$X^n + Y^n = Z^n$ *(01)*

where:

n- prime number, n>2; X,Y,Z are integers.

The solutions of which can be X, Y, Z - relatively prime numbers.

1.Decomposition of (01) into multipliers.

If n is odd, then (01) will decompose into multipliers:

$$X^n + Y^n = (X + Y)(X^{n-1} - X^{n-2}Y + ... - XY^{n-2} + Y^{n-1}) \quad (02)$$

where in the second bracket is the geometric progression

first term $a_1 = X^{n-1}$ *,and a multiplier* $q = -\frac{Y}{X}$

The sum of the members of which $S = \frac{a_1(1-q^n)}{1-q}$

$$Z^n = Z_{11} Z_{22} \quad (03)$$

where:

$$Z_{11} = X + Y \quad (04)$$

$$Z_{22} = X^{n-1} - X^{n-2}Y + ... - X\ Y^{n-1} + Y^{n-1} \quad (05)$$

2.Equivalent representation Z_{22} **.**

If we sum the equidistant terms from the middle term of the progression Z_{22} *in pairs.*

of the middle term of the progression in pairs we have:

for degree 3

$$Z_{22} = (X+Y)^2 - 3XY \quad (06)$$

Fifth degree :

$$Z_{22} = \frac{X^5 + Y^5}{X + Y} = X^4 - X^3\ Y + X^2\ Y^2 - X\ Y^3 + Y^4 \quad (07)$$

$$X^4 + Y^4 = (X + Y)^4 - 4\ X\ Y\ (X + Y)^2 + 2\ X^2\ Y^2 \quad (08)$$

$$-X\ Y^3 - X^3\ Y = -X\ Y\ (X^2 + Y^2) = -X\ Y(X + Y)^2 + 2\ X^2\ Y^2 \quad (09)$$

$$Z_{225} = (X + Y)^4 - 5(X + Y)^2 + 5\ X^2\ Y^2 \quad \text{`}(10)$$

to the 7th degree:

$$Z_{227} = (X + Y)^6 - 7\ X\ Y\ (X + Y)^4 + 14\ X^2\ Y^2\ (X + Y)^2 - 7\ X^3\ Y^3 \quad (11)$$

degree n:

$$Z_{22N} = \frac{X^n + Y^n}{X + Y} =$$

$$= (X+Y)^{n-1} - K_{n-3}XY(X+Y)^{n-3} + ... \mp K_2 X^{\frac{n-3}{2}} Y^{\frac{n-3}{2}} (X+Y)^2 \pm nX^{\frac{n-1}{2}} Y^{\frac{n-1}{2}} \quad (12)$$

$$Z_{22N} = (X+Y)^{n-1} - K_{n-3}XY(X+Y)^{n-3} + ... \mp K_2 X^{\frac{n-3}{2}} Y^{\frac{n-3}{2}} (X+Y)^2 \pm nX^{\frac{n-1}{2}} Y^{\frac{n-1}{2}} \quad (13)$$

where $K_{n-3} ... K_2$ *corresponding coefficients at* $(XY)^{\cdots}(X+Y)^{\cdots}$

equivalent representation Z_{22N} *algebraic sum of even powers of*

X+Y and the residual term $\pm n X^{\frac{n-1}{2}} Y^{\frac{n-1}{2}}$.

Note that by n we mean in <u>*Mathematical deduction1*</u> *any odd power, in* <u>*Mathematical deduction2*</u> *the power of an odd prime number.*

<u>*Mathematical deduction1.*</u>

<u>*Suppose that for an n odd number and for the previous n-2 an equivalent representation(13) is valid, then for the next n+2 it is (13) is valid.*</u>

We show the transition from the two previous odd degrees to the next one further:

$$Z_n^n = X^n + Y^n \quad (14)$$

$$(X^{n-2} + Y^{n-2})(X^2 + Y^2) = X^n + Y^n + Y^2 X^{n-2} + Y^2 Y^{n-2} \quad (15)$$

$$Z_n^n = X^n + Y^n = (X^{n-2} + Y^{n-2})(X^2 + Y^2) - X^2 Y^2 (X^{n-4} + Y^{n-4}) \quad (16)$$

details:

$\frac{X^{n_1} + Y^{n_1}}{X + Y}$ *multiply by* $(X+Y)^2 - 2XY$

$$X^{n+2}+Y^{n+2}=(X+Y)[Z_{22N}(X+Y)^2-2XYZ_{22N}-X^2\ Y^2\ (X^{n-2}+Y^{n-2})]$$ *(17)*

$$(X+Y)^{n-1}-K_{n-3}XY(X+Y)^{n-3}+...\mp K_2\ X^{\frac{n-3}{2}}Y^{\frac{n-3}{2}}(X+Y)^2\pm nX^{\frac{n-1}{2}}Y^{\frac{n-1}{2}}$$ *

* $$(X+Y)^2=(X+Y)^{n+1}-K_{n-1(01)}XY(X+Y)^{n-1}+...\mp K_{4(01)}X^{\frac{n-3}{2}}Y^{\frac{n-3}{2}}\pm nX^{\frac{n-1}{2}}Y^{\frac{n-1}{2}}(X+Y)^2$$

(18)

$$-2\ XY\ [\ (X+Y)^{n-1}-K_{n-3}XY(X+Y)^{n-3}+...\mp K_2\ X^{\frac{n-3}{2}}Y^{\frac{n-3}{2}}(X+Y)^2\pm nX^{\frac{n-1}{2}}Y^{\frac{n-1}{2}}\ ]=$$

$$=$$

$$-2\ XY(X+Y)^{n-1}-K_{n-3(02)}2\ X^2\ Y^2(X+Y)^{n-3}+...\mp K_{2(02)}2\ X^{\frac{n-1}{2}}Y^{\frac{n-1}{2}}(X+Y)^2\pm 2\ X^{\frac{n+1}{2}}Y^{\frac{n+1}{2}}\ n$$

(19)

$$-X^2\ Y^2(X+Y)^{n-3}+K_{n-3(03)}X^3\ Y^3(X+Y)^{n-5}+...\pm K_{2(03)}X^{\frac{n-1}{2}}Y^{\frac{n-1}{2}}(X+Y)^2\mp(n-2)X^{\frac{n+1}{2}}Y^{\frac{n+1}{2}}$$

(20)

where $K_{...(01)}$ *- corresponding coefficients when multiplied by* $(X+Y)^2$,

$K_{...(02)}$ *- corresponding coefficients when multiplied by* $-2XY$,

$K_{...(03)}$ *-corresponding coefficients when multiplied by* $-X^2\ Y^2$

After adding these algebraic terms we again obtain(13)

Mathematical deduction2.

The equivalent representation (13) is valid for any prime n.

By Mathematical deduction1 , if the two previous representations of (13)are valid, of degree 3 and 5, then it is valid for degree 7. Now taking the previous 5 and 7 degrees we have its validity for the 9th degree, etc, which means all odd degrees are described by the above formula.

And since it includes odd prime , it is valid for prime n.

Let us represent (01) as according to Newton's binomial:

$$(X+Y)^n - Z^n = nX^{n-1} Y + \frac{n(n-1)}{2} X^{n-2} Y^2 + ... + \frac{n(n-1)}{2} Y^{n-2} X^2 + n X Y^{n-1} =$$

$$= [(X+Y)-Z]^n - n(X+Y)Z * ... \qquad (21)$$

From (21) it follows that X+Y - Z is divisible by n and further:

$$(X+Y-Z)[(X+Y-Z)^{n-1} - nk_{n-3}(X+Y)Z(X+Y-Z)^{n-3} \pm nk_2(X+Y)^{n-3}Z^{n-3}(X+Y-Z)^2 \mp n(XY)^{\frac{n-1}{2}}]$$

$$= nX^{n-1} Y + \frac{n(n-1)}{2} X^{n-2} Y^2 + ... + \frac{n(n-1)}{2} Y^{n-2} X^2 + n X Y^{n-1} \qquad (22)$$

it follows:

$$(X+Y)[(X+Y)^{n-1} - ... \mp n(X Y)^{\frac{n-1}{2}}] = Z^n \qquad (23)$$

$$Z_{11n} = X+Y \qquad (24)$$

$$Z_{22n} = (X + Y)^{n-1} - nk_{n-3} X Y (X + Y)^{n-3} + ... \pm nk_2 X^{\frac{n-3}{2}} Y^{\frac{n-3}{2}} (X+Y)^2 \mp n X^{\frac{n-1}{2}} Y^{\frac{n-1}{2}} \qquad (25)$$

3.Analysis of Equation (25).

a)From equation (25) $Z_{11} = X+Y$ *and* Z_{22} *cannot have a common factor for except for n.* Z_{22} *consists of members each of which has a factor X+Y, with the exception of the last product* $n X^{\frac{n-1}{2}} Y^{\frac{n-1}{2}}$ *, which in the case of a common factor c X+Y must involve factors of either X and Y, and they are coprime, so (26). From which the following equalities follow in the absence of n:*

$$X+Y = Z_1^n, \qquad Z-X = Y_1^n, \qquad Z-Y = X_1^n \qquad (26)$$

$$Z_{11} = Z_1^n, \quad Z_{22} = Z_2^n, \quad X_{11} = X_1^n, \quad X_{22} = X_2^n, \quad Y_{11} = Y_1^n, \quad Y_{22} = Y_2^n \qquad (27)$$

$$X+Y-Z = n X_1 Y_1 Z_1 K_o \qquad (28)$$

where

K_o *-an integer coprime to the others specified*

except n.

$$Z_1^n = X_1^n + Y_1^n + 2\ n\ X_1\ Y_1\ Z_1\ K_o \qquad (29)$$

$$X - Y = X_1^n - Y_1^n \qquad (30)$$

$$Z_1^n - Z = n\ X_1\ Y_1\ Z_1\ K_o \qquad (31)$$

$$Z_2 = Z_1^{n-1} - n\ X_1\ Y_1\ K_o \qquad (32)$$

$$X - X_1^n = n\ X_1\ Y_1\ Z_1\ K_o \qquad (33)$$

$$X_2 = X_1^{n-1} + n\ Z_1\ Y_1\ K_o \qquad (34)$$

$$Y - Y_1^n = n\ X_1\ Y_1\ Z_1\ K_o \qquad (35)$$

$$Y_2 = Y_1^{n-1} + n\ Z_1\ X_1\ K_o \qquad (36)$$

$$2\ X = Z_1^n - Y_1^n + X_1^n \qquad (37)$$

$$2\ Y = Z_1^n - X_1^n + Y_1^n \qquad (38)$$

$$2\ Z = Z_1^n + X_1^n + Y_1^n \qquad (39)$$

$$Z_1^n - X_1^n - Y_1^n = 2\ n\ X_1\ Y_1\ Z_1\ K_o \qquad (40)$$

$$Z_1^n - [(X_1 + Y_1)^n - n\ X_1^{n-1}\ Y_1 - ... - n\ Y_1^{n-1}\ X_1] = 2\ n\ X_1\ Y_1\ Z_1\ K_o \qquad (41)$$

из чего вытекает:

$Z_1 - X_1 - Y_1 = nK_n$ *from which it follows* $Z_1 > n$ *(42)*

Note X, Y, Z are coprime numbers, as well as $X_{1,} X_{2,} Y_{1,} Y_{2,} Z_{1,} Z_2$.

If the sum or difference of two coprime numbers has a factor n, then

the sum and difference of the n-power of these numbers is divisible by at least n^2 ,

which is obvious from (25), (04).

b) If in the expansion Z, X, Y has a prime factor n

$Z_{22} = nZ_2^n$, $X_{22} = nX_2^n$, $Y_{22} = nY_2^n$ *(43)*

and according to formula (25) Z_2 *cannot have n available, otherwise*

this will lead to the presence of it in X or Y, and vice versa, which is not acceptable.

Z_2, X_2, Y_2 *- does not contain the factor n. In this regard, if Z contains a factor n, then*

formula (26) has the form, since sum $X_1^n+Y_1^n$ *contains a multiplier* n^m *where natural number,* $m\geq 2$

$$n^{nm-1}Z_1^n = X_1^n+Y_1^n+2n^m\ X_1\ Y_1\ Z_1\ K_o \qquad (44)$$

To solve (39) in integers, degree n in $X_1^n+Y_1^n$ *,should be equal*

degree n in the last monomial, that is, minimally n^2 *.*

similar:

$$n^{nm-1}\ X_1^n = Z_1^n - Y_1^n - 2\ n^m\ X_1\ Y_1\ Z_1\ K \qquad (45)$$

$$n^{nm-1}\ Y_1^n = Z_1^n - X_1^n - 2\ n^m\ X_1\ Y_1\ Z_1\ K \qquad (46)$$

$$n^{nm-1}\ Z_1^n - n^m\ Z_{2n}\ Z_1 = n^m\ X_1\ Y_1\ Z_1\ K \qquad (47)$$

$$n^m\ X_{2n}\ X_1 - n^{nm-1}X_1^n = n^m\ X_1\ Y_1\ Z_1\ K \qquad (48)$$

$$n^m\ Y_{2n}\ Y_1 - n^{nm-1}\ Y_1^n = n^m\ X_1\ Y_1\ Z_1\ K \qquad (49)$$

where:

$$Z_2 = n^m Z_{2n},\ Z = Z_1\ Z_2 \qquad (50)$$

$$X_2 = n^m\ X_{2n},\ X = X_1\ X_2 \qquad (51)$$

$$Y_2 = n^m\ Y_{2n},\ Y = Y_1\ Y_2 \qquad (52)$$

Thus

$X+Y-Z = n\ X_1\ Y_1\ Z_1\ K_o$ *universal,*

where $X_{1,}Y_1\ Z_{1,}K_o$ *-coprime corresponds to X,Y,Z with and without n. The difference* *is:*

$K_o = n^{m-1} K$ *(53)*

Since X, Y, Z are relatively prime numbers, the presence of n in one of them obliges the other two to its absence.

4.***Degree n=3***.

According to (33) and Newton's binomial[6]:

$$Z_2^3 = Z_1^6 - 3(X_1^3 + 3X_1Y_1Z_1K_o)(Y_1^3 + 3X_1Y_1Z_1K_o) = (Z_1^2 - 3X_1Y_1K_o)^3 =$$

$$= Z_1^6 - 9Z_1^4X_1Y_1K_o + 27Z_1^2X_1^2Y_1^2K_o^2 - 27X_1^3Y_1^3K_o^3 \quad (54)$$

On the other side :

$$Z_2^3 = (X+Y)^2 - 3XY =$$

$$= Z_1^6 - 3X_1^3Y_1^3 - 9X_1^3X_1Y_1Z_1K_o - 9Y_1^3X_1Y_1Z_1K_o - 27X_1^2Y_1^2Z_1^2K_o^2 \quad (55)$$

Underlined in (55) according to (40):

$$-3(X_1^3 + Y_1^3 - Z_1^3 + Z_1^3)3X_1Y_1Z_1K_o = 2*27X_1^2Y_1^2Z_1^2K_o^2 - 9X_1Y_1Z_1^4K_o \quad (56)$$

$$9K_o^3 = 1, \quad K_o^3 = \frac{1}{9} \quad (57)$$

There is no solution in whole numbers.

If Z contains n:

$$X+Y = 3^{3m-1}Z_1^3 \quad (58)$$

$$3Z_2^3 = 3(3^{3m-1}Z_1^2 - 3^mX_1Y_1K_o)^3 = 3^{9m-2}Z_1^6 - 3^{6m+1}Z_1^4X_1Y_1K_o +$$

$$+ 3^{5m+1}Z_1^2X_1^2Y_1^2K_o^2 - 3^{3m+1}X_1^3Y_1^3K_o^3 \quad (59)$$

$$(X+Y)^2 - 3XY = 3^{6m-2}Z_1^6 - 3XY \quad (60)$$

(59)=(60),when divided by 3^2 *there is no solution in integers.*

And the next solution option according to (25) the difference:

$(X+Y)^{n-1}$ - Z_2^n = $Z_1^{(n-1)n}$ - Z_2^n = $(Z_1^{n-1}-Z_2^n)$ *

* $[(Z_1^{n-1}-Z_2)^{n-1}-...\pm nZ_1^{(n-1)(\frac{n-1}{2})}\ Z_2^{\frac{n-1}{2}}]$ = n^2 * ... *(61)*

The difference factor (54) n is minimal to the second power. Applicable for n=3:

$$Z_1^{2*3}-Z_2^3 = 3\ X_1\ Y_1\ K_o(9\ X_1^2\ Y_1^2\ K_o^2-3\ X\ Y) \quad (62)$$

According to (01):

$$Z_2^3 = (X+Y)^2-3\ X\ Y \quad (63)$$

Hence 3XY is divisible by three squared without remainder, and the need analog X_2 *with monomial 3ZY or* Y_2 *3ZX requires that two relatively prime numbers have a factor n, which does not exist. There is no solution in integers. When the prime factor 3 is contained in the components X, Y, Z, the solution further (79).*

5. Degree *n*.

It is for this case that we examine the balance of the factor n.

Let us consider equation (01) based on (31), (33), (35):

$$(X_1^n+n\ X_1\ Y_1\ Z_1\ K_o)^n+(Y_1^n+n\ X_1\ Y_1\ Z_1\ K_o)^n = (Z_1^n-n\ X_1\ Y_1\ Z_1\ K_o)^n \quad (64)$$

Let's open the brackets:

$$n\ n\ X_1\ Y_1\ Z_1\ K_o\ X_1^{n(n-1)} + n\ n\ X_1\ Y_1\ Z_1\ K_o\ Y_1^{n(n-1)} + n\ n\ X_1\ Y_1\ Z_1\ K_o\ Z_1^{n(n-1)} +...$$

$$...+3*\ (n\ X_1\ Y_1\ Z_1\ K_o)^n + \underline{X_1^{nn}+Y_1^{nn}-Z_1^{nn}} = 0 \quad (65)$$

According to (65), the underlined free term is relatively $n\ X_1\ Y_1\ Z_1\ K_o$ *is divisible without remainder by* n^{m+1} *,where m is the degree taking into account its* K_o *.*

Then we have:

$Z_1^n - X_1^n - Y_1^n$ *(66)*

is divisible without remainder by n^m *and :*

$Z_1^{nn} - X_1^{nn} - Y_1^{nn}$ *(67)*

is divisible without remainder by n^{m+1} *.*

What should we proceed from (25):

$$Z_1^{nn} - X_1^{nn} - Y_1^{nn} = Z_1^{nn} - (X_1^{nn} + Y_1^{nn}) = Z_1^{nn} - (X_1^n + Y_1^n)^n + n(X_1^n + Y_1^n)^{n-2} - \ldots \pm n(X_1\ Y_1)^{\frac{n-1}{2}} *$$

$$* (X_1^n + Y_1^n) =$$

$$= \underline{Z_1^{nn} - (X_1^n + Y_1^n)^n + n\ X_1^n\ Y_1^n (X_1^n + Y_1^n)^{n-2} - \ldots \pm n(X_1^n\ Y_1^n)^{\frac{n-1}{2}} (X_1^n + Y_1^n)} \quad (68)$$

where the sum of the underlined terms, referring to (25) contains n^{m+1} *.*

Further, according to (29):

$$Z_1^{n^2-1} - (Z_1^{n-1} - 2\ n\ X_1\ Y_1\ K_o)(Z_1^n - 2\ n\ X_1\ Y_1\ Z_1\ K_o)^{n-1} + (Z_1^{n-1} - 2\ n\ X_1\ Y_1\ K_o) * n^{m+1} \ldots \quad (69)$$

and then after opening the brackets (other monomials with a factor n^{m+1} *):*

$2\ n\ X_1\ Y_1\ Z_1^{n^2-n}(Z_1^{n-1} - 1)$ *(70)*

inevitably divides into a simple n. From which it follows according to (32) $Z_2^n - 1$ *also divisible without remainder by prime n.*

Similarly:

$X_1[(Z_1^{nn} - Y_1{}^{nn}) - X_1^{nn}]$ *(71)*

according to (29):

$$\frac{Z_1^n - Y_1^n}{X_1} = \frac{X_1^n + 2\ n\ X_1\ Y_1\ Z_1\ K_o}{X_1} = X_1^{n-1} + 2\ n\ Z_1\ Y_1\ K_o \quad (72)$$

further (25):

$$(X_1^{n-1}+2\ n\ Y_1\ Z_1\ K_o)\ (X_1^n+2\ n\ X_1\ Y_1\ Z_1\ K_o)^{n-1}-X_1^{n^2-1} \quad (73)$$

similarly Z_1:

$X_1^{n-1}-1$, $X_2-X_1^{n-1}$, X_2-1 , X_2^n-1 *- are divisible by n (74)*

$Z^{n-1}-1$, $X^{n-1}-1$, $Y^{n-1}-1$ *(75)*

is divisible by prime n without remainder:

$$(X+Y)^{n-1}-1=(X_1^n+Y_1^n+2\ n\ X_1\ Y_1\ Z_1\ K_o)^{n-1}-1 = X_1^{(n-1)n}+...+Y_1^{(n-1)n}-1 \quad (76)$$

$$\frac{(X+Y)^n}{X+Y} - 1 = \frac{(X+Y)^n-(X+Y)}{X+Y} = \frac{(X+Y-Z+Z)^n-(X+Y-Z+Z)}{X+Y} =$$

$$= \frac{...+Z(Z^{n-1}-1)}{X+Y} \quad (77)$$

What follows from this $Z^{n-1}-1$ *is divisible by n without remainder.*

Since (74) are divisible by n, we have:

$X_1^{n-1}-Y_1^{n-1}$, $Z_1^{n-1}-X_1^{n-1}$, $Z_1^{n-1}-Y_1^{n-1}$ *(78)*

$2\ Z_1^{n-1}-X_1^{n-1}-Y_1^{n-1}=n*...$ *(79)*

which shows the impossibility of solving (01) in integers when one of the components X, Y, Z contains a prime n (leads to its appearance in the others).

According to (31), (33), (35):

$Z_1^n-Z_1-Z+Z_1=n\ X_1\ Y_1\ Z_1\ K_o$ *(80)*

From which, obviously, containing one of the three components of a prime n will cause the presence of n in the other two and the solution (01) is absent.

Substitute into (01):

$$(X-X_1+X_1)^n+(Y-Y_1+Y_1)^n=(Z-Z_1+Z_1)^n = ...+Z_1^n-X_1^n-Y_1^n \quad (81)$$

marked by the dotted line, is a minimum multiple without remainder n^2 *since* *the remaining terms divide without remainder by* n^2 *Consider:.*

$$Z_1^n - X_1^n - Y_1^n = 2\ n^2\ X_1\ Y_1\ Z_1\ K \qquad (82)$$

From (75) comes:

$$Z_1^n - Z^n = n^2 * ... \qquad (83)$$

And from (70):

$$Z^{n-1} - 1 = n * ... \qquad (84)$$

Multiplying (84) by Z we get:

$$Z^n - Z = n * ... \qquad (85)$$

Let's add (83)+(85):

$$Z_1^n - Z = n^2\ X_1\ Y_1\ Z_1\ K \qquad (86)$$

Then clearly (75), (84) is divisible without remainder by n^2 *and (86)* $Z - Z_1$ *also contains a multiplier* n^2 *and what leads to integer division by* n^2 *(74), (78), (79). We confirm:*

$$(Z_1^{n-1} - 1 + 1)(Z_2^{n-1} - 1 + 1) - 1 = (Z_1^{n-1} - 1)(Z_2^{n-1} - 1) + \underline{Z_1^{n-1} - 1 + Z_2^{n-1} - 1} = n^2 * ... \qquad (87)$$

The selected number is divided without remainder by n^2 *and add and subtract it from (32) taking into account that in (32) on the right side* n^2 *we have that (74), (78), (79) is divisible*

without a remainder on n^2 *. (87) shown for* n^2 *and is applicable for* n^m *.*

Let us consider the following according to (25):

$$(X + Y)^{n-1} - Z_2^n = (X + Y - Z + Z)^{n-1} - Z_2^n =$$

$$= \underbrace{(X + Y - Z)}_{n^m}[(X + Y - Z)^{n-2} + ... - Z^{n-2}] + \underbrace{Z^{n-1} - 1}_{n^m} \underbrace{- Z_2^n + 1}_{n^{m+1}} = Z_1^{(n-1)n} - Z_2^n = n^{m+1} * ... \qquad (88)$$

From which follows the unique solution for integers, indicated by the line *dashes under the formula:*

$$n^m * ... + n^m * ... - n^{m+1} * ... = n^{m+1} * ... \qquad (89)$$

In case if in $Z^{n-1} - 1$ *does not include the factor n to the power m, then (88) is not solvable in integers relative to prime n.*

$Z - Z_1$, $X - X_1$, $Y - Y_1$ *thus are divided without remainder into* n^m,

and all other members in (81) marked with ellipses begin with n^{m+1} *:*

$$n^{m+1} * \ ...- \ Z_1^n - X_1^n - Y_1^n \ = \ n^{m+1} * \ ...- \ 2 \ n^m \ X_1 \ Y_1 \ Z_1 \ K \ = 0 \qquad (90)$$

We have an imbalance in n and there are no solutions (90), (81). (01) in integers.

6. Conclusion.

If the degree in (01) is odd, there is no solution. Pharm proved the absence of a solution for the 4th degree and thereby proved its absence for everyone $n = 2^m$, *where m is an integer. Fermat's theorem is solvable in the first and second powers!*

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