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Goldbach's problem and twin primes are infinite

Abstract. *The Goldbach-Euler binary problem is formulated as follows: Any even number, starting from 4, can be represented as the sum of two primes. The ternary Goldbach problem is formulated as follows: Every odd number greater than 7 can be represented as the sum of three odd primes, which was finally solved in 2013. [1]-[8].*

In 1995, Olivier Ramare proved that any even number is the sum of no more than 6 primes.[9]

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Theorem*. Difference between any odd number and a prime odd number is equal to any even number and vice versa the difference of any even number and of a prime odd number is equal to any odd number.*

Proof.

$2K+1-p=2N$ *(01)*

where $K=K_{1,}K_1+1,\ldots,K_i=K_1+i-1,\ldots\infty$

$N=N_{1,}N_1+1,\ldots,N_i=N_1+i-1,\ldots\infty$

p is a prime odd number,

j-serial number of a continuous series of natural numbers, starting accordingly with K_1,N_1

K and N are an infinite, continuous series of integers that begin with

K_1, N_1 *, p -any prime number(fixed value, some*

constant). Thus we have (01). And similarly:

$$2N - p = 2K + 1 \qquad (02)$$

the difference of any even and odd numbers andconversely allow to represent any prime odd number.

Corollary1.

If the sum of six primes is any even number, then the sum

primes less than six if odd, any odd number, if even

any even number with corresponding initial values $N_{1,} K_{1.}$

From the equality of the sum of six primes to any even number it follows:

$$p_1 + p_2 + p_3 + p_4 + p_5 + p_6 + 2 = p_7 + p_8 + p_9 + p_{10} + p_{11} + p_{12} \qquad (03)$$

$$p_1 + p_2 + p_3 + p_4 + p_5 + p_6 - p_{12} + 2 = p_7 + p_8 + p_9 + p_{10} + p_{11} \qquad (04)$$

$$2N - p_{12} + 2 = p_7 + p_8 + p_9 + p_{10} + p_{11} \qquad (05)$$

$$2K + 1 = p_7 + \ldots + p_{11} \qquad (06)$$

$$p_7 + p_8 + p_9 + p_{10} + p_{11} + 2 = p_{12} + p_{13} + p_{14} + p_{15} + p_{16} \qquad (07)$$

...

(the index under p is not critical)

$$p_1 + p_2 + p_3 + p_4 + 2 = p_5 + p_6 + p_7 + p_8 \qquad (08)$$

$$p_1 + p_2 + p_3 + p_4 - p_8 + 2 = p_5 + p_6 + p_7 \qquad (09)$$

$$p_5 + p_6 + p_7 = 2K + 1 \qquad (10)$$

where K=3,4,5..., ∞

weak Goldbach problem.

$$p_1+p_2+p_3+2=p_4+p_5+p_6 \quad (11)$$

$$p_1+p_2+p_3-p_4+2=p_5+p_6 \quad (12)$$

$$p_5+p_6=2N \quad (13)$$

where N=2,3,..., ∞

strong Goldbach problem.

Based on the corollary, we solve the following problems.

2.Twin primes are infinite

Corollary2. *Any odd number is the difference between the sum of two prime numbers and a prime number.*

From the solution to the weak Goldbach problem:

$$p_1+p_2+p_3+2=p_4+p_5+p_6 \quad (01)$$

$$p_1+p_2-p_4+2=p_5+p_6-p_3 \quad (02)$$

what needed to be proven.

Theorem2*.A prime number starting at 5 is the arithmetic mean of two simple .*

Let's prove it by contradiction.

$$p_5+p_6\neq 2p_7 \quad (03)$$

where $p_5\neq p_6$

and according Corollary1:

$$p_5+p_6\neq p_1+p_2+p_3+p_4 \quad (04)$$

$$p_5+p_6-p_4\neq p_1+p_2+p_3 \quad (05)$$

which contradicts Corollary2, confirms Theorem2, which confirms the infinity of prime numbers.

Theorem3. *Starting from 14, even numbers are the sum of two odd primes not less than two different representations.*

$p_1+p_2+p_3+p_4=p_1+p_5 = 2N$ *(06)*

$p_2+p_3+p_4=p_5$ *(07)*

Assume by analogy with (07):

$p_1+p_3+p_4=p_6$ *(08)*

add up (07)+(08):

$p_5+p_6=p_1+p_2+2(p_3+p_4)$ *(09)*

according to Corollary1 :

$2(p_3+p_4)=p_7+p_8 = 4\mathrm{N}_j$ *(10)*

where $4\mathrm{N}_j$ *is a fixed even number.*

and

$p_5+p_6=p_1+p_2+p_7+p_8$ *(11)*

and further :

$p_5+p_6-p_7=p_1+p_2+4\mathrm{N}_j-p_7$ *(12)*

$p_1+p_2+2\mathrm{p}_3+2\mathrm{p}_4=p_1+p_2+4\mathrm{N}_j$ *(13)*

and finally:

$p_3+p_4=2\mathrm{N}_j$ *(14)*

corresponds to Corollary1, which confirms Assumption (07).

(07),(08) - inequality in case (14) is not equal to the corresponding certain even number $2\mathrm{N}_j$ *with respect to 2N. However*

redistribution by replacing simple $p_{1,} p_2 , p_3 , p_4$ *we find an even* $2N_j$ *for* $p_1 \neq p_2$ *, which means two representations by the sum of two prime for even 2N.*

Let's say $p_1 = p_2$ *, then* $2p_5 = 2N$ *.Introducing an even through the sum of four simple ones:*

$$p_5 + p_1 + p_3 + p_4 = 2p_5 \qquad (15)$$

$$p_5 = p_1 + p_3 + p_4 \qquad (16)$$

$$p_6 = p_5 + p_3 + p_4 \qquad (17)$$

Thus we have $p_5 \neq p_6$, $p_1 \neq p_2$ *according to Theorem2.*

The second representation would be absent if there were even numbers that cannot be represented as the sum of two prime numbers.

From this follows:

$$p_5 + p_1 = p_6 + p_2 = 2N \qquad (18)$$

where N= 7, 8, 9,10.., ∞

As a result, even numbers starting with 16 are the sum of two prime numbers, at least than two presentations. Up to 16 we determine arithmetically -6,8,12 in one presentations. Hence the values of N.

***Corollary 3**: The number of twins is infinite.*

Corollary 2 is a special case of the above theorem

Let $p_{1,} p_2$ *a pair of twins. Then according to (18)* $p_{5,} p_6$ *inevitably next set of twins. Next, instead of* $p_{1,} p_2$ *, we substitute in (18)* $p_{5,} p_6$ *we have the next pair, etc. So the process is endless and there is no finite pair of twins!*

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