

How to solve this integral?

r, s, o, p, θ are known constants.

The original formulas are found on Page 3 and Page 7 in Cornelissen et al. (2023).

$$g_x = \frac{x-x'}{(x-x')^2+(y-y')^2} \quad (10a)$$

$$\begin{aligned} G_x = \int_r^s \int_{y' \cot \theta}^p g_x dx' dy' = & \frac{y-s}{2} \ln((x-p)^2+(y-s)^2) - \frac{y-r}{2} \ln((x-p)^2+(y-r)^2) \\ & + (x-p) \left(\operatorname{atan} \left(\frac{y-s}{x-p} \right) - \operatorname{atan} \left(\frac{y-r}{x-p} \right) \right) \\ & + \sin^2 \theta \left(\frac{(x-o) \cot \theta + (y-r)}{2} \ln((x-o)^2+(y-r)^2) \right. \\ & \quad \left. - \frac{(x-p) \cot \theta + (y-s)}{2} \ln((x-p)^2+(y-s)^2) \right. \\ & \quad \left. + (x-y \cot \theta) \left(\operatorname{atan} \left(\frac{(x-o) \cot \theta + (y-r)}{x-y \cot \theta} \right) - \operatorname{atan} \left(\frac{(x-p) \cot \theta + (y-s)}{x-y \cot \theta} \right) \right) \right) \end{aligned} \quad (27)$$

Taking derivative of G_x with respect to x yields G_{xx}

$$\begin{aligned} G_{xx} = & \operatorname{atan} \left(\frac{y-s}{x-p} \right) - \operatorname{atan} \left(\frac{y-r}{x-p} \right) + \frac{\sin \theta \cos \theta}{2} \ln \left(\frac{(x-o)^2+(y-r)^2}{(x-p)^2+(y-s)^2} \right) \\ & - \sin^2 \theta \left(\operatorname{atan} \left(\frac{(x-p) \cot \theta + y-s}{x-y \cot \theta} \right) - \operatorname{atan} \left(\frac{(x-o) \cot \theta + y-r}{x-y \cot \theta} \right) \right) \end{aligned} \quad (29)$$

I don't know how $\int_r^s \int_{y' \cot \theta}^p \frac{x-x'}{(x-x')^2+(y-y')^2} dx' y'$ equals Eq. (27) and how $\frac{\partial G_x}{\partial x}$ equals Eq. (29).