A New Fastest Method to Find Perfect Square Roots, Non Perfect Square Roots & Even Higher Roots Like Cube Roots 4^{th}, 5^{th}, 6^{th}, 7^{th} …. n^{th} Roots.

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Introduction-

Perfect square root of any √ X can be describe as x is the square root of y, where x is multiplied twice to get y.

For example square root of 9 is 3.

$3^2 = 9$.

In case of cube & higher roots, that the x is multiplied more than twice to get y

For example,

Cube root of 27 is 3, $3^3 = 27$

Fourth root of 81 is 3, $3^4 = 81$.

A non perfect square root of √ X is that every Integer that is not the result of squaring an integer with itself.

E.g.,√ X are 5, 7, 10, 21

There are various method of finding square roots like Initial estimate, Babylonian method, Bakhshali method, Newtons Method. Many of those method are about guessing method with trial and error. Some of them take longer time to compute with vast series of calculations.

I introduce a new method which is simple, yet fastest method to calculate with a 100% accuracy to find perfect square roots and with better approximation in case of non prefect square roots of any given √X. Also using this method one can easily find higher roots like cube roots, 4^{th}, 5^{th}, 6^{th}, 7^{th} …. n^{th} roots.

Formula -

$$\frac{x}{72} = m$$

Below is the simple formula which we will use to find 'Key Integer' to find m which is required for calculating c.

1.1) Solving Problems of √x Where, x Has a Decimals Fractions –

Note -

We can to use this method only where fractions decimal numbers occurs at √x

This method can also use for square or any non square roots including higher roots.

E.g., 9.9 × 9.9 = 98.01, √x = 98.01

× 7 = 6.93, √x = 6.93

A)Taking x as Positive Integer by Removing Decimal Point -

This step is very crucial in finding square roots, cube roots or even of higher roots like 4^{th}, 5^{th}, 6^{th} roots. Where, √x is decimal fractions.

One must completely neglect and take the x as positive integer without decimal place.

E.g.,

If problem to solve √x are,

√245.67 then take as x = 24567

√235.93562 then take as x = 23593562

Adding Zero -

In case if we are finding non square roots then, after removing decimal point remember to add 0 at Ones place.
E.g. of √78.4
√7840
7840 will be used as dividend to calculate at formula x / 72 = m
Check solved example at - Second Example Finding Non Perfect Square Root of √ x, Where x Has a Fraction Decimals..

Rest is to follow all the same steps, as we follow to find square roots including higher roots explained in this paper. (For better understanding check below **First Examples** at both problems on finding square roots and higher roots).

B) **Placing of Decimal Point-**

As explained above we were in need of removing the decimal point for the sake of finding solution.
In the same way, we need to place it back again at the Key Integer c. This step is required in order to check whether Key Integer c is our final answer or not.
Suppose while finding for √ x = we got c = 1116

1^{st} E.g.,
Suppose while finding for √ x = 126.1129 we got c = 1116
Here at this point we must convert 1116 in decimals and check whether c is the final answer by dividing x by c.
Now the problem is, we'll definitely not know where to place the decimal point next to which digit. Therefore we will place decimal point after every digit of 1116 and check ti divide it with x.

E.g.,
c = 1.116
c = 11.16
So, we get 11.16 as divisor.
 Dividing and checking √ x = 126.1129 by c = 11.16
Therefore, we found 11.16 is our answer.

2^{nd} E.g.,
While solving some problem, example we get c = 39
Since it has only two digits, in such case we can place decimal point in between two digits 3.9

We need this method of checking to be applied at 4^{th} and 5^{th} steps to find the solution. As shown below at explanation -- 'First Example – Finding Square Root of √ x, Where √ x Has Fractions Decimals '.

1.2) Repeated Subtraction of Series –

To find the key integer 'c' we will use the below **repeated subtraction series** where we will subtract every difference of series using the following **Sequence Series** -

Repeated Subtraction Series

$$m_1 - 1 = m_2$$
$$m_2 - 2 = m_3$$
$$m_3 - 3 = m_4$$

$$Eg.$$
$$m_1 = 5$$
$$5 - 1 = 4$$
$$4 - 2 = 2$$

Img No.1 Img No.2

1) While calculating **repeated subtraction series**, note that each difference of the series can be strictly 0 or any positive integers. E.g as shown at the above image no. 1 the difference are positive.
i.e. 5-1 = **4**, 4 - 2 = **2**

2 – 3 = -1 (series must not have negative difference)

Note - Always use below any one of the two methods to find Key Integer ‘c’.

<u>1.3) First Method of Finding Key Integer ‘c’ - Repeated Subtraction Series.</u>

$$m_1 - 1 = m_2$$
$$m_2 - 2 = m_3$$
$$m_3 - 3 = m_4$$

Img 1.3

$$Eg.$$
$$m_1 = 5$$
$$5 - 1 = 4$$
$$4 - 2 = 2$$

Img 1.4

1) As we find m from the formula X / 72= m where X is 360 / 72 = 5
Take m as m_1 = 5
Substituting in the ‘Repeated Subtraction Series’ as shown below.
5 -1 = 4
4 - 2 = 2

2 – 3 = -1 (continuing from above when we subtract 2 by 3 we get -1).

But as stated above at explanation 1.1, we cannot have negative difference.
This way one can go on calculating until the series reaches the point where we will start getting negative difference as result. Once we reach at this point of the series where we start getting negative results, we must stop the calculation. Above shown example series ends at 4 - 2 = 2. so,we must stop calculating series at this point.

2) While Finding Key Integer c we use x formula
x / 72 = m …..(is a quotient) those quotient is subtracted in sequence to get last negative integer and that last negative integer is our Key Integer c.

At above example we got -2 as Key Integer c.

3) We must always convert the negative key integer to positive. E.g, **Key Integer -2** will be 2 as positive integer. Conversion using simple equation - a × -1 = c, here as per the example a = -2
Therefore, -2 × -1 = 2

At above example, we got the last series as 4 -2 = 2
Now, notice the last subtracting sequence term that is -2
So, we must consider the negative integer -2 as positive integer. So, we get **Key Integer 'c'.** Therefore**, c = 2** as positive integer.

1.3 A) Fixed value of Key Integer c -

While finding Key Integer c, if we find the value of Key Integer c = 0 then always take c = 6
E.g., To find √ 25
We use formula 25/ 72 = 0.347… (ignore decimals)
As we further calculate using repeated subtraction series to find key integer c by, we get c = 0

Here, since we get 0 value therefore for further calculation we must take fixed value as c = 6

This fixed value can be taken especially while finding certain √ x as explained below -
√ 2 /72 = 0.027.... As per the method take directly c = 6 and as per sub rule (at 1.1 B) take 6 – 4 = 1, 2 / 1 = 2 so our answer for non perfect square root is√ 2 = 1 approx.

√ 3 /72 = 0.0416.... As per the method take directly c = 6 and as per sub rule (at 1.1 B) take 6 – 4 = 2, 3 / 2 = 1.5 so our answer for non perfect square root of √ 3 = 1.5 approx.

√ 5 /72 = 0.0694.... As per the method take directly c = 6 and as per sub rule (at 1.1 B) take 6 – 4 = 2, 5 / 2 = 2.5 so our answer for non perfect square root of √ 5 = 2.5 approx.

√ 7 /72 = 0.0972.... As per the method take directly c = 6 and as per sub rule (at 1.1 B) take 6 – 3 = 3, 7 / 3 = 2.3333333 So our answer for non perfect square root of √ 7 = 2.3333333 approx.

Check below more example solution at – '<u>Seventh Example, Finding Perfect Square Root of √X'.</u>

<u>1.4) Second Method of Finding Key Integer 'c' – Successive Series</u>.

Choosing Second Successive **Series-**
Note – Always remember to take second series in case of finding key integer c as in case it appears at both place.

e.g. If you notice below series, 3 appears twice in two series simultaneously. So as we need to find key integer c and we got two options to choose from then, always take the second series

e,g,
1 --- 3 has only as difference of one (neglect this first series)
3 ---- 6 has a difference of two (take consider this second series)

Successive Series -

0 -- 1 has zero difference
1 --- 3 has only as difference of one i.e. 2
3 ---- 6 has a difference of two i.e. 4 and 5
6 ---- 10 has a difference of three i.e. 7 to 9
10 ---- 15 has a difference of four i.e. 11 to 14
15 ---- 21 has a difference of five i.e. 16 to 20
21 ---- 28 has a difference of six i.e. 22 to 27
28 ---- 36 has a difference of seven i.e. 28 to 35
36 ---- 45 has a difference of eight i.e. 37 to 44
45 ---- 55 has a difference of nine i.e. 46 to 54
55 ---- 66 has a difference of ten i.e. 56 to 65
66 ---- 78 has a difference of eleven i.e. 67 to 77
78 ---- 91 has a difference of twelve i.e. 79 to 90
91 ---- 105 has a difference of thirteen i.e. 92 to 104
105 ---- 120 has a difference of fourteen i.e. 106 to 119
120 ---- 136 has a difference of fifteen i.e. 121 to 135
136 ---- 153 has a difference of sixteen i.e. 137 to 152

This series is a form of successive series where sequence of numbers on the right hand side that is, 3, 6, 10, 15, 21….
Is the result of addition of sequence of positive integers such as 1, 2, 3, 4, 5, 6, 7, 8….
E.g., 1 + **2** = 3, 3 + **3** = 6, 6 + 4 = 10 ….

As show above, one can continue create series by adding 153 + 18 = 171, 171 + 19 = 190…..record and use it whenever required to calculate Key Integer ‘c'. Series can go to infinity . This second method is very useful when we need to find Larger Key Integer ‘c’ of any larger √ X.

Example - Let suppose we want to find key integer of √ 2209

$$\frac{x}{72} = m$$ Using

√ 2209 / 72 = 30.68056

Ignoring the decimals, we got m = 30

To find key integer from m = 30 , check the above ‘Pattern Series’ at 1.3

30 appears between this part of above series at 28 --30-- 36 and has a difference of seven i.e. 28 to 35

n.

Therefore, we must take Key Integer ‘c'

c = 7

<u>1.5) Calculation of Steps For Higher Roots –</u>

Note – The below conditions must be only followed in case while finding Key Integer ‘c' of any higher roots like cube roots, 4^{th} , 5^{th} , 6^{th}, 7^{th} roots…..

For Cube root – up to 2 steps.

For 4^{th} root – up to 3 steps.

For 5^{th} root – up to 4 steps

Fir 6^{th} root – up to 5 steps

For 7^{th} roots up to 6 steps (and so on increase one step for every nth roots)

As higher roots increases, the steps of calculation increase by one as shown at the above. Use the applicable steps depending on which roots you want to find.

Example- we want to find **Key Integer ‘ c’** of cube root 3 √1331.

Check the required steps of calculation for cube root at above calculation steps explanation 1.4
Therefore, for cube root, we must calculate up till 2 steps of calculation.

As usual first find m using formula.
$\sqrt{}$ 1331 / 72 = 18.486...
Ignoring the decimals, we got m = 18

Following 1st step -
To find key integer from m = 18 , check the above ‘Pattern Series’ at 1.3
18 somewhere appears between this part of above series 15 --18-- 21 and has a difference of **five** i.e. from 16 to 20

Therefore, we must take Key Integer ‘c'
c = 5
Following 2st step -
Carried forward c = 5 from 1st step and substitute it with m
To find key integer from m = 5 , check the above ‘Pattern Series’ at 1.3
5 somewhere appears between this part of above series 3 ---- 6 and has a difference of **two** i.e. 4 and 5

Therefore, we must take Key Integer ‘c'
c = 2

This way one can easily find the Key Integer ‘c' using the above explained method.

1.6) Multiplying c by Constant 6 -

This step is very important and is calculated after we get Key Integer value ‘c'
E.g., at above explanation of calculation of steps at 1.4 we get c = 2

$2 \times 6 = 12$

Now we get c = 12

Very Important to Note - That in some case of finding solutions we may get c itself as final answer. We can check c by dividing it with associated √ X . If it is divisible, and we get square roots solutions as required, then c is the final answer, if it doesn't yields solution then proceed to below rules to find final answer c (for better understanding check various examples solutions shown at this paper).

1.7) Rules of Finding c In case of Square Roots -

As we find the final answer ‘c as square root through the formula we must consider the following rule that is required to get the perfect square root with 100% accurately and in case of non perfect square root we can get answer to some degree of accuracy.

First Rule - If the √ X is any odd integer and is not divisible by 2 or 3, then the final answer will be either adding or c by 1 i.e., c +1, adding c by 5 i.e. c + 5 adding c by 7 i.e. c + 7

Second Rule - If the√ X is any even integer and is divisible by 2 or 3, then the final answer will be by either adding c by 3 i.e. ‘c + 3’ or adding c by 6 i.e. ‘c + 6’.

Third Rule- If the √ X is any even integer which is divisible by 2, then the final answer will be either adding c by 4 i.e. ‘c + 4 or adding c by 8 i.e. ‘c + 8'.

Fourth Rule - If the √ X is any odd integer and is divisible by 3, then the final answer c will be adding c by 3 i.e. ‘c + 3'.

1.7 A) Sub Rule For Finding √ X.

Where, √ X = r , where r is the square roots.

Due to change in order of operations, we require this additional sub rule to be applied for all those r > 5

Rule - If the √ X is any odd or even integer then the final answer c will be subtracting c by 1, 2 ,3, 4 i.e. c.- 1, c.- 2, c.-3, c.- 4.

Check example solution below at - **Seventh Example – Finding Perfect Square Root of √X.**

To find square root of any given √ X integer we must follow all the above explained formula, series and rules to get the precise results. No matter how larger digit the √ X integers would be, using the method explained in this paper, one can get the answer very quickly and precisely.

Below Are Some of The Examples Solutions of Finding Square Roots –

First Example – Finding Perfect Square Root of √ x, Where x Has Decimal Place Value.

1st Step- Taking x as Positive Integer by Neglecting Decimal Point.

To find perfect square root of √98.01
We take √98.01 as √9801 by neglecting decimal point, as explained at 1.1
To find square root we will use x = 9801

2st Step - Using the below formula by substituting 9801 at x to get m.

$$\frac{x}{72} = m$$

9801 / 72 = 136.125   … ignore decimals

m_1 = 136

We will take m value as m_1 to substitute it in the below repeated subtraction series.

3nd Step – (Finding The Key Integer ‘c’ Using First Method- Repeated Subtraction Series.   ….. from 1.2

Note that just for explanation purpose, I have shown both example of finding Key Integer c. One can use any of one of the two method for finding Key Integer c).

$$m_1 - 1 = m_2$$

$$m_2 - 2 = m_3$$

$$m_3 - 3 = m_4$$

By substituting the m_1 = 2 in the repeated subtraction series we get,

Where m_1 = 136
136 - 1 = 135
135 - 2 = 133

133 - 3 = 130
130 - 4 = 126
126 - 5 = 121
121 – 6 = 115
115 - 7 = 108
108 - 8 = 100
100 - 9 = 91
91 - 10 = 81
81 - 11 = 70
70 - 12 = 58
58 – 13 = 45
45 – 14 = 31
31 – 15 = 16
16 – 16 = 0

As per the ‘condition of series’ we cannot further calculate 0 - 17 = -17 since it gives negative value.
Therefore we stop the calculation at 16 – 16 = 0
Last negative series of sequence is -16 and by converting it as positive integer, -16 × -1 = 16
we got **Key Integer ‘c’ as** 16
Therefore, c =16

Explanation of Using Second Method to Find Key Integer ‘c’ …… from 1.3

To find key integer c of √ 9801

$$\frac{x}{72} = m$$

using
√ 9801 / 72 = 136.125 ... ignore decimals

m = 136

To find key integer from m = 2 , check the above 'Successive Series'.
136 appears at the above successive series somewhere between 136 ---- 153 and has a difference of **sixteen.**

Therefore, we must take **Key Integer 'c'**
c = 16

<u>4th Step –</u> Multiplying c by Constant 6 & Placing of Decimal Point.

Therefore, 16 × 6 = 6
we get c = 96
Here at this point we must convert 96 in fraction decimals and check whether c is final answer. Therefore follow the method of adding decimal point as explained above at 1.1 B
In this case, we will place decimal point at 9.6
So, we get 9.6 as divisor.
Dividing and checking 98.01 by 9.6

We found it is not divisible.
Therefore, we will proceed to below 5^{th} step of checking **Rules of Finding 'c.**

<u>5th Step –</u> Checking the **Rules of Finding 'c'** to find perfect square root.
c = 96

√X = 98.01 and we took √X = 9801

√X = 9801 is an odd integer divisible by 3. Therefore it follows the fourth rule of finding 'c.

Fourth Rule - If the √ X is any odd integer and is divisible by 3, then the final answer c will be adding c by 3 i.e. 'c + 3'.
Checking both options by substituting value of c.

$96 + 3 = 99$

<u>Important Step- Placing of Decimal Point of Key Integer c</u>

Here at this point we must convert 96 in decimal and check whether c is final answer. Therefore follow the method explained above at 1.1 B

In this case, we will place decimal point at 9.9

We get c = 9.9

$\sqrt{98.01}$ is divisible by 9.9

Therefore,

So, $\sqrt{98.01} = 9.9$

<u>Second Example – Finding Non Perfect Square Root of √ x, Where x has a Decimal Fractions .</u>

1st Step- Taking X as Positive Integer by Removing Decimal Point.

To find perfect square root of $\sqrt{78.4}$

We take $\sqrt{78.4}$ as $\sqrt{78.4}$ by removing decimal point, as explained above at 1.1 A

To find square root, we will use x = 784

We will add 0 having its place value as Ones. E.g., 7840 ……..as explained above at 1.1 A - Adding Zero.

<u>2st Step -</u> Using the below formula by substituting 7840 at x to get m.

$$\frac{x}{72} = m$$

7840 / 72 = 108.888889 … (ignore decimals)

m_1 = 108

We will take m value as m_1 to substitute it in the below repeated subtraction series.

3nd Step – (Finding The Key Integer ‘c’ Using First Method- Repeated Subtraction Series. ….. from 1.2

Note that just for explanation purpose, I have shown both example of finding Key Integer c. One can use any of one of the two method for finding Key Integer c).

$$m_1 - 1 = m_2$$

$$m_2 - 2 = m_3$$

$$m_3 - 3 = m_4$$

By substituting the m_1 = 108 in the repeated subtraction series we get,

Where m_1 = 108
108 - 1 = 107
107 - 2 = 105
105 - 3 = 102
102 - 4 = 98
98 - 5 = 93
93 – 6 = 87

87 – 7 = 80
80 - 8 = 72
72 – 9 = 63
63 – 10 = 53
53 – 11 = 42
42 – 12 = 30
30 – 13 = 17
17 – 14 = 3

As per the ‘condition of series’ we cannot further calculate 3 - 15 = -12 since it gives negative value.
Therefore we stop the calculation at 17 – 14 = 3
Last negative series of sequence is -14 and by converting it as positive integer, we got **Key Integer ‘c’ as** 14
Therefore, c =14

4th Step – Multiplying c by constant 6.
Therefore, 14 × 6 = 84
we get c = 84

In the case of non perfect square will proceed directly to 5rd step of checking **Rules of Finding ‘c.**

5th Step – Checking the **Rules of Finding ‘c’** to find square root.
c = 84

√X = 78.4 and we took √X = 7840

√X = 7840 is an even integer divisible by 2. Therefore it follows the third rule of finding ‘c.

Third Rule- If the √ X is any even integer which is divisible by 2, then the final answer will be either adding c by 4 i.e. ‘c + 4 or adding c by 8 i.e. ‘c + 8'.

Checking both options by substituting value of c.
84 + 2 = 86
84 + 4 = 88
84 + 8 = 92
Here at this point we must convert all options in decimals and check whether c is final answer. Therefore follow the method explained above at 1.1 B
In this case, we will place decimal point at 8.6, 8.8, 9.2

Therefore,
78.4 / 8.8 = 8.90909090
So, √78.4 = 8.90909090 .... close aproximation.

Third Example – Finding Perfect Square Root of √ x, Where x has a Decimal Fractions.

1st Step- Taking x as Positive Integer by Removing Decimal Point.

To find perfect square root of √126.1129
We take √ 126.1129 as √1261129 by neglecting decimal point, as explained above at 1.1
To find square root, we will use x = 1261129

2st Step - Using the below formula by substituting 1261129 at x to get m.

$$\frac{x}{72} = m$$

1261129 / 72 = 17515.681 … ignore decimals

$m_1 = 17515$

We will take m value as m_1 to substitute it in the below repeated subtraction series.

3^nd Step – (Finding The Key Integer ‘c’ Using First Method- Repeated Subtraction Series. ….. from 1.2

Note that just for explanation purpose, I have shown both example of finding Key Integer c. One can use any of one of the two method for finding Key Integer c).

$$m_1 - 1 = m_2$$

$$m_2 - 2 = m_3$$

$$m_3 - 3 = m_4$$

By substituting the $m_1 = 2$ in the repeated subtraction series we get,

Where $m_1 = 17515$
17515 - 1 = 17514
17514 - 2 = 17512
17512 - 3 = 17509
17509 - 4 = 17505
17505 - 5 = 17500
.

.

Note – Due to less space,I haven’t showed above, the full counting of repeated subtraction series. One can continue calculating by following repeated subtraction series and find **Key Integer ‘c’ as to be** 186
Therefore, c = 186

<u>4th Step – </u>Multiplying c by constant 6 & Placing of Decimal Points.

Therefore, 186 × 6 = 1116
we get c = 1116
Here at this point we must convert 1116 in decimal fractions and check whether c is final answer. Therefore follow the method adding decimal point as explained above at 1.1 B
In this case, we will place decimal point next to every digits.
1.116
11.16

Dividing and checking 98.01 by 1.116 and 11.116
We found it is not divisible.
Therefore, we will proceed to below 5rd step of checking **Rules of Finding ‘c.**

<u>5th Step – </u>Checking the **Rules of Finding ‘c’** to find perfect square root.
c = 1116

√X = 1261129 is an odd integer not divisible by 2 or 3. Therefore it follows the first rule of finding ‘c.
First Rule - If the √ X is any odd integer and is not divisible by 2 or 3, then the final answer will be either adding c by 5 i.e. c + 5 and adding c by 7 i.e. c + 7

Checking both options by substituting value of c.
1116 + 5 = 1121
1116 + 7 = 1123
Important Step- Placing of Decimal Point of Key Integer c.

Now, adding the decimals at both options.
Here at this point we must convert 1121 and 1123 in fraction decimal and check whether c is final answer. Therefore following the method explained above at 1.1 B
In this case, we will place decimal point next to every digits.
1.121, 11.21, 1.123, 11.23

Out of all options we found $\sqrt{126.1129}$ is divisible by 11.23
Therefore,
So, $\sqrt{126.1129} = 11.23$

Fourth Example –

To find perfect square root of $\sqrt{196} = s$, where s is the square root.

1^{st} Step - Using the below formula by inputting 196 to get m.

$$\frac{x}{72} = m$$

196/ 72 = 2.7222 …..
By ignoring the decimals, we get m = 2
We will take m value as m_1 to input it in the below repeated subtraction series.
$m_1 = 2$

2^{nd} Step – (Finding The Key Integer ‘c’ Using First Method- Repeated Subtraction Series. ….. from above 1.3

Note that just for explanation purpose, I have shown both example of finding Key Integer c at some of the example solutions. One can use any of one of the two method for finding Key Integer c).

$$m_1 - 1 = m_2$$

$$m_2 - 2 = m_3$$

$$m_3 - 3 = m_4$$

By substituting the $m_1 = 2$ in the repeated subtraction series we get,

Where $m_1 = 2$
2 -1 = 1

As per the ‘condition of series’ we cannot further calculate 1- 2 = -1 since it gives negative value.
Therefore we stop the calculation at 2 – 1 = 1
Last negative series of sequence is -1 and by converting it as positive integer, -1 × -1 = 1
we got **Key Integer ‘c’ as** 1
Therefore, c =1

<u>Explanation of Using Second Method to Finding Key Integer ‘c’</u> …… from 1.3

To find key integer c of √ 196

$$\frac{x}{72} = m$$ using

$\sqrt{}$ 196 / 72 = 2. 72222...
Ignoring the decimals, we got m = 2
To find key integer from m = 2 , check the above 'Successive Series at 1.4
2 appears between this part of above successive series somewhere between 1 --- 3 and this series has a difference of one.
Therefore, we must take **Key Integer 'c'**
c = 1

<u>2th Step –</u> Multiplying c by constant 6.

Therefore, 1× 6 = 6
Now we get c = 6
Checking whether c is final answer by dividing 196 by 6. We found it is not divisible.
Therefore, we will proceed to below 3rd step of checking **Rules of Finding 'c.**

<u>3th Step –</u> Checking the **Rules of Finding 'c'** to find precise perfect square root of c = 6

$\sqrt{}$X = 196 is an even integer divisible by 2. Therefore it follows the third rule of finding 'c.
Third rule states - If the $\sqrt{}$ X is any even integer which is divisible by 2, then the final answer will be either adding c by 4 i.e. 'c + 4 or adding s by 8 i.e. 'c + 8'
Checking both options by substituting value of c at all options.
6 + 4 = 10
6 + 4 = 14
Since 196 is divisible by 14
Therefore, our required answer is 6 + 4 = 14
So, $\sqrt{196}$ = 14

Fifth Example –

To find perfect square root of √3249

1st Step - Using the formula by substituting 3249 at X to get m.

$$\frac{x}{72} = m$$

3249 / 72 = 45.125 …..
By ignoring the decimals, we get m = 45
We will take m value as m_1 to input it in the below repeated subtraction series.
m_1 = 45

2nd Step – Finding the Key Integer ‘c’ Using repeated subtraction series. ……..from 1.2
By substituting the m_1 = 45 in the series we get,

Where m_1 = 45
45 - 1 = 1
44 - 2 = 42
42 - 3 = 39
39 - 4 = 35
35 - 5 = 30
30 - 6 = 24
24 - 7 = 17
17 - 8 = 9
9 - 9 = 0

As per the ‘condition of series’ we cannot further subtract 0 – 10 = -10 since it gives negative value.
Therefore we stop the calculation at 9 – 9 = 0
Now, the last negative series of sequence at the above series is -9 and by converting it to positive integer, -9 × -1 = 9
we get **Key Integer ‘c’ as 9**
Therefore, c = 9

Explanation of Using Second Method to Finding Key Integer ‘c’ …… from 1.3
3249 / 72 = 45.125 …..
By ignoring the decimals, we get m = 45

To find key integer from m = 45 , check the above ‘Successive Series’.
45 appears at above successive series that is at 45 ---- 55 and has a difference of nine.

Therefore, we must take **Key Integer ‘c'**
c = 9

2th Step – Multiplying c by constant 6.

Therefore, 9 × 6 = 54
Now we get c = 54
Checking whether c is final answer by dividing 3249 by 54 . We found it is not divisible.
Therefore, we will proceed to below 3rd step of checking **Rules of Finding ‘c.**

3th Step – Checking the **Rules of Finding ‘c’** to find precise perfect square root of c = 54

$\sqrt{X} = \sqrt{3249}$ is divisible integer divisible by 3. Therefore it follows the fourth rule of finding c.

Fourth Rule - If the √ X is any odd integer and is divisible by 3, then the final answer c will be adding s by 3 i.e. 'c + 3'.

54 + 3 = 57

Since 3249 is divisible by 57
Therefore, √3249 = 57

Fifth Example –

To find perfect square root of √529

1st Step - Using the formula by substituting 529 at x to get m.

529/ 72 = 7.347…
Ignore the decimals, we get m = 7
We will take m value as m_1 and substitute it in the below repeated subtraction series.
$m_1 = 7$

2nd Step – Finding the Key Integer 'c' Using repeated subtraction series.
By substituting $m_1 = 7$ in the series we get,

Where $m_1 = 7$
7 -1 = 6
6 -2 = 4
4 -3 = 1

As per the ‘condition of series’ we cannot further subtract 1 – 4 = -3 since it gives negative value.

Therefore we stop the calculation at 4 – 3 = 1

Now, the last negative series of sequence at the above series is -3 and by taking it as positive integer, we get **Key Integer ‘c’ as 3.**

9 × -1 = 9

Therefore, c = 3

2th Step – Multiplying c by constant 6.

Therefore, 3 × 6 = 18

Now we get c = 18

Checking whether c is final answer by dividing 529 by 18 . We found it is not divisible.

Therefore, we will proceed to below 3rd step of checking **Rules of Finding ‘c.**

3th Step – Checking the **Rules of Finding ‘c’** to find precise perfect square root of c = 18

√X = √529 is an integer not divisible by 2 or 3. Therefore it follows the first rule of finding c.

First Rule - If the √ X is any odd integer and is not divisible by 2 or 3, then the final answer will be either adding c by 5 i.e. c + 5 adding c by 7 i.e. c + 7

18 + 5 = 23

18 + 7 = 25

Since 529 is divisible by 23 therefore, our answer is at 18 + 5 = 23

Therefore, √3249 = 23

<u>Sixth Example (Non Perfect Square Root) –</u>

Note – in the case of finding non perfect square root of any integer √X we can get only the better approximation with decimals.
To find non perfect square root of √1817
<u>1st Step -</u> Using the formula by substituting 1817 to X to get m.

$$\frac{x}{72} = m$$

1817 / 72 = 25.26111
Ignore the decimals, we get m = 25
We will take m value as m_1 to input it in the below repeated subtraction series.
m_1 = 25

<u>2nd Step –</u> Finding the **Key Integer 'c' Using repeated subtraction series.**
By inputting the m_1 =25 in the series we get,

Where m_1 = 25
25 -1 = 24
24 -2 = 22
22 -3 = 20
20 -4 = 16
16 -5 = 11
11 -6 = 5

As per the 'condition of series' we cannot further subtract 5 - 7 = -2 since it will give negative value as -2

Therefore we stop the calculation at 11 – 6 = 5
Now, the last negative series of sequence at the above series is -6 and by taking it as positive integer, we get **Key Integer 'c' as 6**
Therefore, c = 6

2^{th} Step – Multiplying c by constant 6.

Therefore, 6 × 6 = 36
Now we get c = 36

Checking whether c is final answer by dividing 1817 by 36 . We found it is not divisible.
Therefore, we will proceed to below 3^{rd} step of checking **Rules of Finding 'c.**

3^{th} Step – Checking the **Rules of Finding 'c'** to find precise perfect square root of c = 36

√X = 1817 is an odd integer not divisible by 3. Therefore it follows the first rule of finding c.
First Rule - If the √ X is any odd integer and is not divisible by 2 or 3, then the final answer will be either adding c by 5 i.e. c + 5 adding c by 7 i.e. c + 7

Checking both options by substituting value of c at all options.
36 + 5 = 41
36 + 7 = 43
The close approximation of 1817 / 43 = 42.2558139
Therefore, our required answer is at 36 + 7 = 43

So, √1817 = 43

Finding Square Roots of √x .

In this ploblems we will use **sub rule** stated at - 1.7 A) Sub Rule For Finding √ X = r. where r > 5
e.g., √16 = 4 .... 4 is less than 5.

Seventh Example – Finding Perfect Square Root of √X

To find perfect square root of √16

1st Step - Using the formula by substuting 16 at x to get m.

$$\frac{x}{72} = m$$

16 / 72 = 0.2222222..
Ignore the decimals, we get m = 0
We will take m value as m_1 to substitute it in the below repeated subtraction series as
$m_1 = 0$

2nd Step – Finding the **Key Integer 'c' Using repeated subtraction series.**
By inputting the m_1 =0 in the series we get,

Where m_1 = 0
0 -1 = -1

As per the ‘condition of series’ at the first step itself we got 0 -1 = -1 and since it gives negative value as -1 therefore, this step of series is negligible. Therefore we stop the calculation at first step and the **Key Integer ‘c’** will be 0

Therefore, c = 0

<u>3th Step –</u> Using c = 6 as stated at 1.3 A.

Since we get c = 0 therefore, we will take c = 6 fir further calculation.

Checking whether c is final answer by dividing 16 by 6

We found 16 is not divisible by 6.

Therefore, we will proceed to below 4rd step of checking **Rules of Finding ‘c.**

<u>4th Step –</u> Checking the **Rules of Finding ‘c’** to find precise perfect square root.

√X = 16 is an even integer and has Key Integer c = 6

Therefore, we will follow rule explained at - **1.7 A) Sub Rule For Finding √ X.**

Rule - If the √ X is any odd or even integer then the final answer c will be any of the either solution.

subtracting c by 1, 2 ,3, 4, 5, 7 i.e. c.- 1, c.- 2, c.- 3, c.- 4.

After checking all options, we found 6 – 2 = 4 to be our required answer.

Therefore,

So, √16 = 4

Finding The Cube Roots & Higher n^{th} Roots Like 4^{th}, 5^{th} ,6^{th} ..... Roots of Given √X .

In the case of finding higher roots we need to take slightly different approach such as using rules explained below at 2.1, steps of ‘Repeated Subtraction Series’ or ‘Successive Series’ to get the final result.

2. 1) Rule of finding ‘c’ (to find final answer) -

First rule - If the √X is any odd integer and is not divisible by 2 or 3, then we must take any of the solutions - adding c by 1 i.e. c+1, subtracting c by 1 i.e. c -1, substracting c by 5 i.e. c - 5 substract c by 7 i.e. c - 7

Second Rule - If the √X is any even integer and is divisible by 2 or 3, then we can take any of the solutions - subtracting c by 3, 6 or 9 i.e. ‘c - 3’, ‘c - 6’, ‘c - 9’.
Third Rule - If the √ X is any even integer which is divisible by 2, then we can take any of the solutions - subtracting c by 2, 4, 8 or 10 i.e. ‘c - 2', ‘c - 4', ‘c - 8', ‘c - 10'.

Fourth Rule - If the √ X is any odd integer and is divisible by 3, then we can take any of the solutions - subtracting c by 3, 6 or 9 i.e. ‘c - 3’, ‘c - 6’, ‘c - 9

(Important Note- In case of finding higher roots like cube roots, 4^{th} , 5^{th} , 6^{th} roots…..

Rules of addition or subtraction of c might get increase to c – 12, c – 14 etc…but I am not sure though. One can calculate higher roots and check whether such problem appears and where we require such solutions like c – 12, c – 15, c – 16 …..c - n or we don't require, at any of the above associated stated rules).

2.2) Steps of Repeated Subtraction Series For Each Higher Roots -

Cube root – up till 2 steps.
4th root – up till 3 steps.
5th root – up till 4 steps
6th root – up till 5 steps
7th roots up till 6 steps and so on …..
As higher roots increases, the steps of calculating repeated subtraction series increase by one as shown at the above.

Note - In some case while calculatng repeated subtraction series before following all the required steps we may land up at - 1 as result. In such case it is not need to get through all steps and -1 of that step will be the our required answer for further finding of Key Integer c.

e.g. To find fourth root of $\sqrt{625}$ we need to follow up to 3 steps.

625 /72 = 8.68.....
in this case following the substraction series, we get only two steps of calculation.
at 1st step we get - 2 at the end of series.
at 2nd step we get - 1 at the end of series.
We cannot calculate 3rd step, therefore, in such case we can consider -1 from 2nd step, as the last step required result and contunue finding Key Integer c.

Below Are The Example of Solutions –

First Example – Finding Cube Root of √ X, Where X is a Decimals Fraction.

1st Step- Taking X as Positive Integer by Removing Decimal Point.

To find cube root of √10.648
We take √10.648 as √10648 by neglecting decimal point, as explained above at 1.1
To find square root, we will use x = 10648

2st Step - Using the below formula by substituting 10648 at x to get m.

$$\frac{x}{72} = m$$

10648 / 72 = 147.88889 … (ignore decimals)

m_1 = 147

We will take m value as m_1 to substitute it in the below repeated subtraction series.

3nd Step – (Finding The Key Integer ‘c’ Using First Method- Repeated Subtraction Series. ….. from 1.3

Note that just for explanation purpose, I have shown both example of finding Key Integer c. One can use any of one of the two method for finding Key Integer c).

$m_1 - 1 = m_2$

$m_2 - 2 = m_3$

$m_3 - 3 = m_4$

Since we are finding cube root, therefore we will calculate Repeated Subtraction Series up to second step.

<u>First Step of Calculation (Repeated Subtraction Series). -</u>

By substituting the $m_1 = 147$ in the repeated subtraction series,

Where $m_1 = 147$
147 - 1 = 146
146 - 2 = 144
144 - 3 = 141
141 - 4 = 137
137 - 5 = 132
132 - 6 = 126
126 - 7 = 119
119 - 8 = 111
111 - 9 = 102
102 - 10 = 92
92 - 11 = 81
81 - 12 = 69
69 - 13 = 56
56 - 14 = 42
42 - 15 = 27

27 - 16 = 11

As per the ‘condition of series’ we cannot further calculate 11 - 17 = -6 since it gives negative value.
Therefore we stop the calculation at 27 - 16 = 11
Now, the last negative series of sequence at the above series is - 16 and by taking it as positive integer, - 16 × 16 = 16
we get **Key Integer ‘c’ as 16**

Second Step of Calculation (Repeated Subtraction Series). -

To calculate second step, carry forward the **Key Integer c = 16** from the above first step series. Again here $m_1 = 16$
Calculating again the below **repeated subtraction series.**

16 - 1 = 15
15 - 2 = 13
13 - 3 = 10
10 - 4 = 6
6 - 5 = 1

As per the ‘condition of series’ we cannot further calculate as 1 - 6 = - 5 since it gives negative value. Therefore, we stop the calculation at 6 -5 = 1

Now, the last final negative series of sequence at the above series is -2 and by taking it as positive integer, we get **Key Integer ‘c’ as 5, c = 5**

Explanation of Using Second Method to Finding Key Integer ‘c’ …… from 1.3
(Note that just for explanation purpose, i have shown both example of finding Key Integer c. One can use any of one, out of the two method for finding Key Integer c).

As per the condition stated at ‘ **1.4 - Steps For Each Higher Roots’** we must calculate up to two steps for cube root finding.

1st Step of Calculation -

10648 / 72 = 147.88889 … (ignore decimals)

m_1 = 147

Ignore the decimals, we get m = 147

To find key integer from m = 18 , check the above ‘Successive Series’.
5 appears at above successive series somewhere between 136 ---- 153 and has a difference of sixteen.

Therefore, we take **Key Integer ‘c'**
c = 16

2nd Step of Calculation -

Carried forward c = 16 from ‘1st step of calculation’.
To find key integer from 16, again check the above ‘Successive Series’.
5 appears at above successive series somewhere between 15 ---- 21 and has a difference of five.

Therefore, we must take **Key Integer ‘c'**
c = 5

<u>4th Step –</u> Multiplying c by constant 6 & Placing of Decimal Point.

c = 5

Therefore, 5 × 6 = 30
we get c = 30

Here at this point we must convert 30 in decimals and check whether c is final answer.
…….. as stated at above 1.1 B adding of decimal point.

To do, we will place decimal point in between 30 and check which one divides x = √10.648
We get 3.0 as divisor.
Dividing and checking 10.648 by 3.0
We found it is not divisible.
Therefore, we will proceed to below 5rd step of checking **Rules of Finding ‘c.**

5th Step – Checking the **Rules of Finding ‘c’** to find perfect square root.
c = 30
√ X = 10.648 we took X = 10648
X = 10648 is an even integer divisible by 2. Therefore it follows the third rule of finding ‘c.
Third Rule - If the √ X is any even integer which is divisible by 2, then we must do subtraction c by 2, 4, 8 or 10 i.e. ‘c - 2', ‘c - 4', ‘c - 8', ‘c - 10'.

Checking all options by substituting value of c.
30 – 2 = 28
30 – 4 = 26
30 – 8 = 22
30 – 10 = 20

Important Step- Placing of Decimal Point.

To check which above options is our answer, therefore we will place decimal point in between 28, 26, 22, 20

So, we get 2 8, 2 6, 2.2, 2.0 as divisors. …… as stated at 1.1 B adding of decimal point.
Here out of above all options we find √10.648 is divisible by 2.2
Therefore,

$$\sqrt[3]{10.648}$$

= 2.2

Second Example-

Let suppose we want to find cube root of

$$\sqrt[3]{1331}$$

1st Step - Using the below formula by substituting 1331 at X to get m.

$$\frac{x}{72} = m$$

1331/ 72 = 18.486111
Ignore the decimals, we get m = 18
We will take m value as m_1 to input it in the below repeated subtraction series.
m_1 = 18

2nd Step – Finding the **Key Integer 'c' Using repeated subtraction series.**
By inputting the m_1 = 18 in the series we get,
In this case of finding cube root we must to calculate **repeated subtraction series.**
up to two steps.

First Step of Calculation (Repeated Subtraction Series). -

Where m_1 = 18
18 -1 = 17
17 -2 = 15
15 -3 = 12
12 - 4 = 8
8 - 5 = 3

As per the 'condition of series' we cannot further 3 – 6 = -3 since it gives negative value.
Therefore we stop the calculation at 8 – 5 = 3
Now, the last negative series of sequence at the above series is - 5 and by taking it as positive integer, -5× -5 = 5
we get **Key Integer 'c' as 5**

Second Step of Calculation (Repeated Subtraction Series). -

To calculate second step, carry forward the **Key Integer c = 5**
from the above first step series. Again here m_1 = 5
Calculating again the below **repeated subtraction series.**

5 -1 = 4
4 -2 = 2

As per the 'condition of series' we cannot further 2 -3 = -1
since it gives negative value.
Therefore we stop the calculation at 4 -2 = 2

Now, the last final negative series of sequence at the above series is -2 and by taking it as positive integer, we get **Key Integer 'c' as 2, c = 2**

Explanation of Using Second Method to Finding Key Integer 'c' …… from 1.3

(Note that just for explanation purpose, I have shown both example of finding Key Integer c. One can use any of one, out of the two method for finding Key Integer c).

As per the condition stated at ‘ **1.4 - Steps For Each Higher Roots’**
we must calculate up to two steps for cube root finding.

1st Step of Calculation -

1331/ 72 = 18.486111

Ignore the decimals, we get m = 18

To find key integer from m = 18 , check the above ‘Progressive Series’.

18 appears at above pattern series that is 15 --18-- 21 and has a difference of five.

Therefore, we must take **Key Integer ‘c'**

c = 5

2nd Step of Calculation -

Carried forward c = 5 from ‘1st step of calculation’.

To find key integer from 5, check the above ‘Successive Series’.

5 appears at above successive series somewhere between 3 ---- 6 and has a difference of two.

Therefore, we must take **Key Integer ‘c'**

c = 2

3th Step – Multiplying c by constant 6.

Therefore, $2 \times 6 = 12$

Now we get c = 12

Checking whether c is final answer by dividing 1331 by 12 . We found it is not divisible.

Therefore, we will proceed to below 4^{rd} step of checking **Rules of Finding 'c.**

<u>4.th Step –</u> Checking the **Rules of Finding 'c'** to find precise perfect square root.

$\sqrt{X} = 1331$ is an odd integer not divisible by 2 or 3. Therefore it follows the first rule of finding c.

First rule - If the $\sqrt{X}$ is any odd integer and is not divisible by 2 or 3, then we must do subtraction c by 1, 5 or 7 i.e. 'c - 1', 'c - 5', 'c - 7'.

Checking by substituting value of c at all options.

12 – 1 = 11 .

12 - 5 = 7

12 - 7 = 5

Therefore, our required answer is at 12 - 1 = 11

So,

$\sqrt[3]{1331}$

= 11

<u>Third Example-</u>

To find cube root

$\sqrt[3]{5832}$

<u>1^{st} Step -</u> Using the below formula by substituting 5832 at x to get m.

$$\frac{x}{72} = m$$

5832 / 72 = 81

We get m = 81

We will take m value and substitute it to m_1 in the below repeated subtraction series.
$m_1 = 81$
<u>**2^{nd} Step –**</u> Finding the **Key Integer 'c' Using repeated subtraction series.**
$m_1 = 81$
In this case of finding cube root we must calculate the same repeated subtraction series up to two steps as shown below.
<u>**First Step of Calculation (Repeated Subtraction Series). -**</u>

Where $m_1 = 81$
81 - 1 = 80
80 - 2 = 78
78 - 3 = 75
75 - 4 = 71
71 - 5 = 66
66 - 6 = 60
60 – 7 = 53
53 – 8 = 45
45 – 9 = 36
36 – 10 = 26
26 – 11 = 15
15 – 12 = 3

As per the 'condition of series' we cannot further calculate 3 – 13 = -10 since it gives negative value.
Therefore we stop the calculation at 15 – 12 = 3
Now, the last negative series of sequence at the above series is - 12 and by taking it as positive integer, -12× -1 = 12
we got **Key Integer 'c' as 12**

<u>**Second Step of Calculation (Repeated Subtraction Series). -**</u>

To calculate second step, carry forward the **Key Integer c = 12**

from the above first step series. Again here m_1 = 12
Calculating again the below **repeated subtraction series.**

12 -1 = 11
11- 2 = 9
9 - 3 = 6
6 – 4 = 2

As per the 'condition of series' we cannot further calculate 2 -3 = -1
since it gives negative value.
Therefore we stop the calculation at 6 - 4 = 2

Now, the last final negative series of sequence at the above series is - 4 and by taking it as positive integer, we get **Key Integer 'c' as 4, c = 4**

<u>Explanation of Using Second Method to Finding Key Integer 'c'</u>
…… from 1.3
As per the condition stated at ' **1.4 - Steps For Each Higher Roots'**
we must calculate up to two steps for cube root finding.
1st step of calculation -

5832 / 72 = 81 ....Ignore the decimals, we got m = 81
To find key integer from 81 , check the above 'Successive Series'.
81 appears at above successive series somewhere between i.e. 78 --81-- 91 and has a difference of twelve.

Therefore, we must take **Key Integer 'c'**
c = 12
2nd Step of Calculation -
Carried forward c = 12 from '1st step of calculation'.
To find key integer from 12 m, check the above 'Successive Series'.
12 appears at above successive series somewhere between 10 --12---- 15 and has a difference of four.

Therefore, we must take **Key Integer 'c'** c = 4

Next,

<u>3^{th} Step –</u> Multiplying c by constant 6.

Therefore, $4 \times 6 = 24$

Now we get c = 24

Checking whether c is final answer by dividing 5832 by 24 . We found it is divisible but it is not our final square roots answer.

Therefore, we will proceed to below 4^{rd} step of checking **Rules of Finding 'c.**

<u>4^{th} Step –</u> Checking the **Rules of Finding 'c'** to find precise perfect square root of c = 24

$\sqrt{X}$ = 5832 is an even integer divisible both by 2 and 3 . Therefore it follows the second rule of finding c.

Second Rule - If the $\sqrt{X}$ is any even integer and is divisible by 2 or 3, then we must do subtraction c by 3, 6 or 9 i.e. 'c - 3', 'c - 6', 'c - 9'.

Checking by substituting value of c at all options.

24 – 3 = 21

24 - 6 = **18**

24 – 9 = 15

Therefore, our required answer is at 24 - 6 = 18

So,

$$\sqrt[3]{5832}$$

= 18

<u>Fourth Example-</u>

To find fourth root of

$\sqrt[4]{10000}$

<u>1st Step -</u> Using the below formula by inputting 10000 to get m.

$$\frac{x}{72} = m$$

10000/ 72 = 138.88889

Ignoring decimals we got,

m =138

We will take m value as m_1 to substitute it in the below repeated subtraction series.

m_1 = 138

<u>2nd Step –</u> Finding the **Key Integer 'c' Using repeated subtraction series.**

As per the condition stated at ' Steps of Repeated Subtraction Series'. In this case of finding fourth root we must calculate the same repeated subtraction series up to three steps as shown below.

inputting the m_1 = 138 in the series.

<u>First Step of Calculation (Repeated Subtraction Series). -</u>

Where m_1 = 138

138 - 1 = 137

137 - 2 = 135

135 - 3 = 132

132 - 4 = 128

128 - 5 = 123

123 - 6 = 117

117 – 7 = 110

110 – 8 = 102
102 – 9 = 93
93 – 10 = 83
83 – 11 = 72
72 – 12 = 60
60 – 13 = 47
47 – 14 = 33
33 – 15 = 18
18 – 16 = 2

As per the 'rule of series' we cannot further calculate 2 – 17 = -15 since it gives negative value.
Therefore we stop the calculation at 18 – 16 = 2
Now, the last negative series of sequence at the above series is - 16 and by taking it as positive integer, -16 × -1 = 16
we get **Key Integer 'c' as 16**

<u>Second Step of Calculation (Repeated Subtraction Series). -</u>

To calculate second step, carry forward the **Key Integer c = 16**
from the above first step series. Again here m_1 = 16
Calculating again the below **repeated subtraction series.**

16 -1 = 15
15 - 2 = 13
13 - 3 = 10
10 – 4 = 6
6 – 5 = 1

As per the 'condition of series' we cannot further subtract 1 - 6 = -5
since it gives negative value.
Therefore we stop the calculation at 6 - 5 = 1

Now, the last negative series of sequence at the above series is - 5 and by taking it as positive integer, -5 × -1 = 5
we get **Key Integer 'c' = 5**

<u>Third Step of Calculation (Repeated Subtraction Series). -</u>

To calculate third step, carry forward the **Key Integer c = 5**
from the above first step series. Here m_1 = 5
Calculating again the below **repeated subtraction series.**

5 -1 = 4
4 - 2 = 2

As per the 'condition of series' we cannot further subtract 2 - 3 = -1 since it gives negative value.
Therefore we stop the calculation at 4 - 2 = 2
Now, the last final negative series of sequence at the above series is - 2 and by taking it as positive integer, -2 × -1 = 2
We get **Key Integer 'c' as 2, c = 2**

<u>Explanation by Using Second Method to Find Key Integer 'c'</u> …… from 1.3
As per the condition stated at ' **1.4 - Steps For Each Higher Roots'**
we must calculate up to three steps for fourth root finding.

1st step of calculation -

Finding m using formula,

10000 / 72 = 81.88...

Ignore the decimals, we got m = 138

To find key integer from 138, check the above 'Pattern Series'.

138 appears at above pattern series somewhere between i.e. 136 -138----- 153 and has a difference of sixteen.
Therefore, we must take **Key Integer 'c'**

c = 16

2^{nd} Step of Calculation -

Carried forward c = 16 from ‘1^{st} step of calculation’.

To find key integer from 16, check the above ‘Pattern Series’.

12 appears at above pattern series somewhere between 15 --16---- 21 and has a difference of five.

Therefore, we must take **Key Integer ‘c'**

c = 5

Next,

3^{rd} Step of Calculation -

Carried forward c = 5 from ‘2^{nd} step of calculation’.

To find key integer from 5, check the above ‘Pattern Series’.

12 appears at above pattern series somewhere between 3 --5---- 6 and has a difference of two.

Therefore, we must take **Key Integer ‘c'**

c = 2

Next,

3^{th} Step – Multiplying c by constant 6.

Therefore, 2 × 6 = 12

Now we get c = 12

4^{th} Step – Checking the **Rules of Finding ‘c’** to find precise perfect square root of c = 12

√X = 10000 is an even integer divisible by 2. Therefore it follows the third rule of finding c.

Third Rule - If the √ X is any even integer which is divisible by 2, then we must do subtraction c by 2, 4, 8 or 10 i.e. ‘c - 2', ‘c - 4', ‘c - 8', ‘c - 10'.

Checking by substituting value of c at all options.

12 – 2 = 10
12 - 4 = 8
12 – 8 = 4
12 – 10 = 2

Therefore, our required answer is at 12 – 2 = 10

So,

$$\sqrt[4]{10000}$$

= 10

Fifth Example-

To find cube root of

$$\sqrt[3]{4913}$$

1st Step - Using the below formula by inputting 4913 to get m.

$$\frac{x}{72} = m$$

4913 / 72 = 67.236111

Ignoring decimals we get whole number as

m = 67

We will take m value as m_1 to input it in the below repeated subtraction series.

m_1 = 67

2nd Step – Finding the **Key Integer 'c' Using repeated subtraction series.**

As per condition stated at 2.3 - ‘Steps of Repeated Subtraction Series’, in this case of finding third root we must calculate the same **repeated subtraction series.**
up to two steps as shown below.
Substituting the m_1 = 67 in the series.

First Step of Calculation (Repeated Subtraction Series). -

Where m_1 = 68
68 - 1 = 67
67 - 2 = 65
65 - 3 = 62
62 - 4 = 58
58 - 5 = 53
53 - 6 = 47
47 – 7 = 40
40 – 8 = 32
32 – 9 = 23
23 – 10 = 13
13 – 11 = 2

As per the ‘condition of series explained at 1.1’ we cannot further calculate 2 – 12 = -10 since it gives negative value.
Therefore we stop the calculation at 13 – 11 = 2
Now, the last negative series of sequence at the above series is - 11 and by taking it as positive integer, -11 × -1 = 11
we get **Key Integer ‘c’ as 11**

Second Step of Calculation (Repeated Subtraction Series). -

To calculate second step, carry forward the **Key Integer c = 11**
from the above first step series. Again here m_1 = 11
Calculating again the below **repeated subtraction series.**

11 -1 = 10

$10 - 2 = 8$
$8 - 3 = 5$
$5 – 4 = 3$

As per the 'condition of series explained at 1.1' we cannot further subtract $3 – 5 = - 2$ since it gives negative value.
Therefore we stop the calculation at $5 - 4 = 3$
Now, the last negative series of sequence at the above series is – 4 and by taking it as positive integer, $-4 \times -1 = 4$
we got **Key Integer 'c' = 4**

3th Step – Multiplying c by constant 6.

Therefore, $4 \times 6 = 24$
Now we get $c = 24$

4th Step – Checking the **Rules of Finding 'c'** to find precise perfect square root of $c = 24$

$\sqrt{X} = 4913$ is an odd integer not divisible by 2 or 3 Therefore it follows the first rule of finding c.

First rule - If the $\sqrt{X}$ is any odd integer and is not divisible by 2 or 3, then we must do subtraction c by 1, 5 or 7 i.e. 'c - 1', 'c - 5', 'c - 7'.

Checking by substituting value of c at all options.
$24 – 1 = 28$
$24 - 5 = 19$
$24 – 7 = 17$

Therefore, our required answer is at $24 – 7 = 17$
So,
$\sqrt[3]{4913}$

= 17

Sixth Example -

To find fourth root of

$$\sqrt[4]{14641}$$

1st Step - Using the below formula by substituting 14641 at X to get m.

$$\frac{x}{72} = m$$

14641 / 72 = 203.34722

Ignoring decimals we got, m = 203

We will take m value as m_1 to substitute it in the below repeated subtraction series.

m_1 = 203

2nd Step – Finding the **Key Integer 'c' Using repeated subtraction series.**

As per the condition stated at ' 2.3 - Steps of Repeated Subtraction Series', so in this case of finding fourth root we must calculate the same **repeated subtraction series** up to three steps as shown below.

inputting the m_1 = 203 in the series.

First Step of Calculation (Repeated Subtraction Series). -

Where m_1 = 203

203 - 1 = 202

202 - 2 = 200

200 - 3 = 197
187 - 4 = 193
193 - 5 = 188
187 - 6 = 182
182 – 7 = 175
175 – 8 = 167
167 – 9 = 158
158 – 10 = 148
148 – 11 = 137
137 – 12 = 125
125 – 13 = 112
112 – 14 = 98
98 – 15 = 83
83 – 16 = 67
67 – 17 = 50
50 – 18 = 32
32 – 19 = 13

As per the ‘condition of series’ we cannot further calculate 13 – 20 = -7 since it gives negative value.
Therefore we stop the calculation at 32 – 19 = 13
Now, the last negative series of sequence at the above series is - 19 and by taking it as positive integer, -19 × -1 = 19
we get **Key Integer ‘c’ as 19**

Second Step of Calculation (Repeated Subtraction Series). -

To calculate second step, carry forward the **Key Integer c = 19**
from the above first step series. Again here m_1 = 19
Calculating again the below **repeated subtraction series.**

19 -1 = 18
18 - 2 = 16
16 - 3 = 13

13 – 4 = 9
9 – 5 = 4

As per the ‘condition of series’ we cannot further subtract 4 - 6 = - 2 since it gives negative value.
Therefore we stop the calculation at 9 - 5 = 4
Now, the last negative series of sequence at the above series is - 5 and by taking it as positive integer, -5 × -1 = 5
we get **Key Integer ‘c’ = 5**

<u>Third Step of Calculation (Repeated Subtraction Series). -</u>

To calculate third step, carry forward the **Key Integer c = 5** from the above first step series. Here m_1 = 5
Calculating again the below **repeated subtraction series.**

5 -1 = 4
4 - 2 = 2

As per the ‘condition of series’ we cannot further subtract 2 - 3 = -1 since it gives negative value.
Therefore we stop the calculation at 4 - 2 = 2
Now, the last final negative series of sequence at the above series is - 2 and by taking it as positive integer, -2 × -1 = 2
We get **Key Integer ‘c’ as 2, c = 2**

<u>3th Step –</u> Multiplying c by constant 6.

Therefore, 2 × 6 = 12
Now we get c = 12

<u>4th Step –</u> Checking the **Rules of Finding ‘c’** to find perfect square root of c = 12

$\sqrt{X}$ = 14641 is an odd integer not divisible by 2 or 3 Therefore it follows the first rule of finding c.

First rule - If the $\sqrt{X}$ is any odd integer and is not divisible by 2 or 3, then we must do subtraction c by 1, 5 or 7 i.e. 'c - 1', 'c - 5', 'c - 7'.

Checking by substituting value of c at all options.

12 – 1 = 11

12 - 5 = 7

12 – 7 = 5

Therefore, our required answer is at 12 – 1 = 11

So,

$\sqrt[4]{14641}$

= 11

<u>Seventh Example –</u>

To find fourth root of

$\sqrt[4]{256}$

<u>1st Step -</u> Using the formula by substituting 256 at x to get m.

$$\frac{x}{72} = m$$

256 / 72 = 3.5555556

Ignore the decimals, we get m = 3

We will take m value as m_1 to substitute it in the below repeated subtraction series as

m_1 = 3

2nd Step – Finding the **Key Integer ‘c’ Using repeated subtraction series.**

As per the condition stated at ‘ 2.3 - Steps of Repeated Subtraction Series', so in this case of finding fourth root we must calculate the same **repeated subtraction series** up to three steps as shown below.
inputting the $m_1 = 3$ in the series.
First Step of Calculation (Repeated Subtraction Series). -

By substituting $m_1 = 0$ in the series we get,

1st Step of Calculation -
Where $m_1 = 3$
3 -1 = 2
3 – 2 = 1
….. we cannot further calculate this series; therefore

We get Key Integer c = 2

2nd Step of Calculation –
Carrying forward c = 2 from 1st step.
Now $m_1 = 2$
2 - 1 = 1
Now here in this case we cannot further calculate from this point,
we get c = 1

As per the ‘condition of series’ at the first step itself we get 1 -2 = -1 and since it gives negative value as -1 therefore, this step of series is negligible. Therefore we stop the calculation at first step and the **Key Integer ‘c’** will be 0
Therefore, c = 1

<u>3^{th} Step –</u> Multiplying c by constant 6.

In this case since we got c = 1, we will add c by 6
1 × 6 = 6
c = 6

Checking whether c is final answer by dividing16 by 6
We found 16 is not divisible by 6.
Therefore, we will proceed to below 4^{rd} step of checking **Rules of Finding ‘c.**

<u>4^{th} Step –</u> Checking the **Rules of Finding ‘c’** to find perfect square root.

√X = 256 is an even integer divisible by 2, therefore it follows the third rule of finding c.

Third Rule - If the √ X is any even integer which is divisible by 2, then we must do subtraction c by 2, 4, 8 or 10 i.e. ‘c - 2', ‘c - 4', ‘c - 8', ‘c - 10'.

Checking by substituting value of c at options.
6 – 2 = 4
6 - 4 = 2
Therefore, our required answer is at 6 – 2 = 4

So,

Therefore,

So, √256 = 4

<u>Finding p and q the multiples of 'n', using the explain method and formulas -</u>

(Note – this method may also work for $p \times q = n$ up till p and q has a gap of eight. But I am not sure whether the solution will be correct or not, but it works perfectly for $p \times q = n$, where p and q has a gap of 2, as explained below).

The formula method explained in this paper also works for any given $p \times q = n$
Where, p and q integers has a gap of two.

E.g, Twin Prime Numbers are the set of two integers that have exactly one composite number between them, therefore having gap of two between two primes.

p q (twin primes has gap of two). p q (two integers having gap of two).

5 7	10 12
11 13	24 25

Using the explained formulas we can also find n of those multiplied $p \times q = n$
E.g, finding solution of 65 where its multiplied p and q has a gap of 8.
In this paper we will consider only about twin primes.

<u>First Example –</u>

To find p and q of given n = 143

1st Step - Using the formula by inputting 143 to get m.

$$\frac{x}{72} = m$$

143 / 72 = 1.9861111
Ignore the decimals, we get m = 1
We will take m value as m_1 to input it in the below repeated subtraction series.
$m_1 = 1$

2nd Step – Finding the **Key Integer 'c' Using repeated subtraction series.**
By substituting the $m_1 = 1$ in the series we get,

Where $m_1 = 1$
1 -1 = 0

As per the 'condition of series' we cannot further 0 – 2 = -2 since it gives negative value.
Therefore we stop the calculation at 1 – 1 = 0
Now, the last negative series of sequence at the above series is – 1 and by taking it as positive integer, we get **Key Integer 'c' as 1**

3th Step – Multiplying c by constant 6.

Therefore, 1 × 6 = 6
Now we get c = 6

4th Step – Checking the **Rules of Finding 'c'** to find precise perfect square root of c = 6

$\sqrt{X}$ = 143 is an odd integer not divisible by 2 or 3 therefore it follows the first rule of finding c.

Note – In the case of finding solutions to such problems always take stated rules at – '1.2) Rules of finding 'c'' for square roots.

So in this case we will take below rule -

First rule - If the $\sqrt{X}$ is any odd integer and is not divisible by 2 or 3, then we must add c by 1, 5 or 7 i.e. 'c - 1', 'c - 5', 'c - 7'.

Checking by substituting value of c at all options.
6 + 1 = 7
6 + 5 = 11
6 + 7 = 13
Therefore, our required answer is at 6 + 5 = 11
6 + 7 = 13

So,
143 is a multiple of 11× 13

Second Example -

To find p and q of given n = 323

1st Step - Using the formula by inputting 323 to get m.

323 / 72 = 4.486…
Ignore the decimals, we get m = 4
We will take m value as m_1 to substitute it in the below repeated subtraction series.
$m_1 = 1$

2nd Step – Finding the **Key Integer 'c' Using repeated subtraction series.**
By substituting the $m_1 = 4$ in the series we get,

Where $m_1 = 4$
4 -1 = 3
3 - 2 = 1

As per the 'condition of series' we cannot further 1 – 3 = - 2 since it gives negative value.
Therefore we stop the calculation at 3 – 2 = 1
Now, the last negative series of sequence at the above series is – 2 and by taking it as positive integer, we get **Key Integer 'c' c= 2**

3th Step – Multiplying c by constant 6.

Therefore, 2 × 6 = 12
Now we get c = 12

4th Step – Checking the **Rules of Finding 'c'** to find precise perfect square root of c = 12

√X = 323 is an odd integer not divisible by 2 or 3 therefore it follows the first rule of finding c.

Note – in the case of finding solutions to such problems always take - 1.2 rules of finding ‘c’ for square roots

So in this case we will take below rule -

First rule - If the √X is any odd integer and is not divisible by 2 or 3, then we must do subtraction c by 1, 5 or 7 i.e. ‘c - 1’, ‘c - 5’, ‘c - 7’.

Checking by substituting value of c at all options.
12 + 1 = 13
12 + 5 = 17
12 + 7 = 19
Therefore, our required answer is at 12 + 5 = 17
12 + 7 = 19

So,
323 is a multiple of 17 and 19

Reference –

https://en.m.wikipedia.org/wiki/Methods_of_computing_square_roots