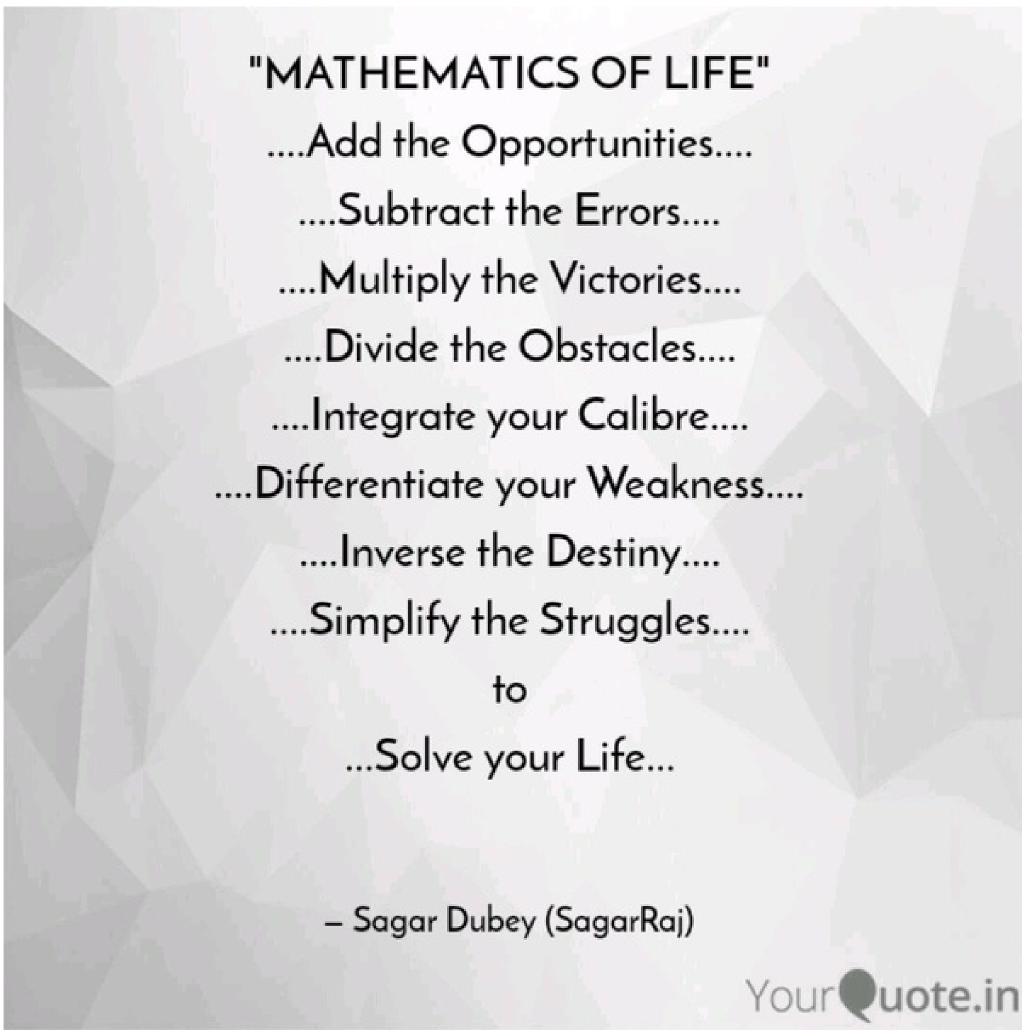
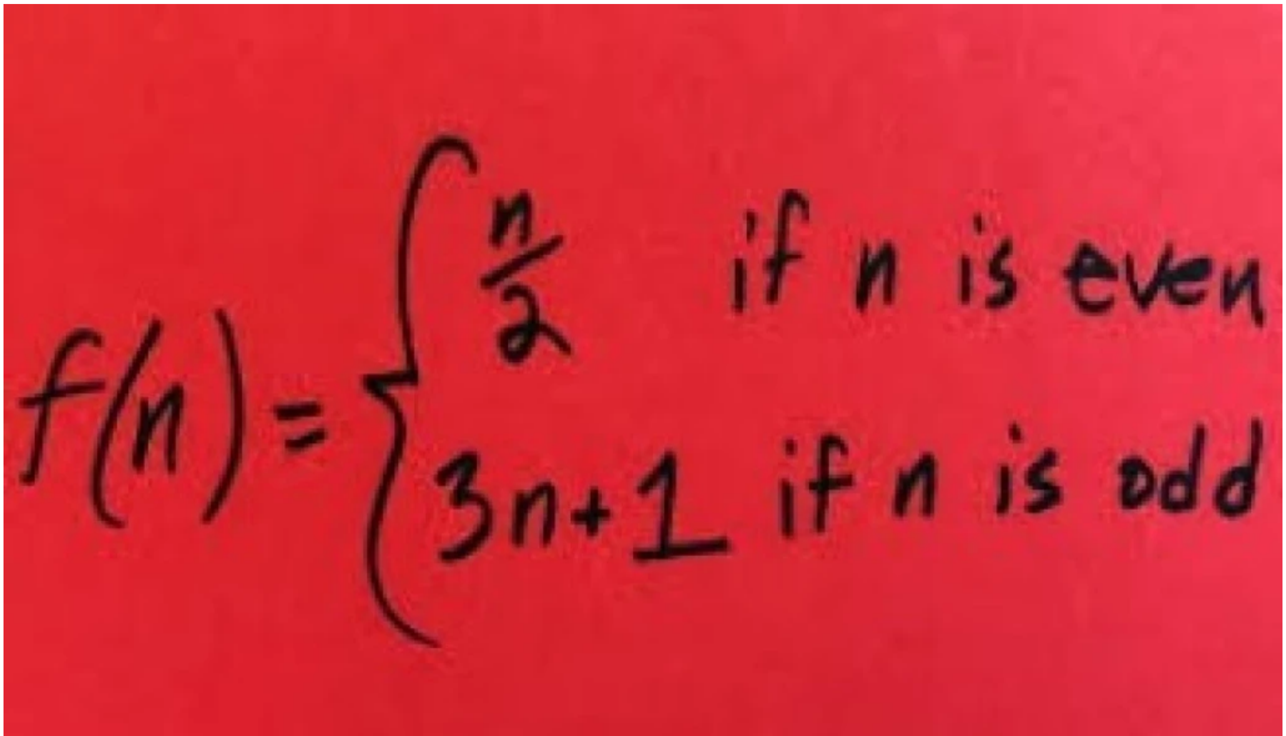
A Brief Analysis of the Collatz Conjecture

Dave September 17, 2020
Collatz Conjecture, Math Conjectures, Number Theory, Poetry, The Art of Problem Solving
Collatz Conjecture, Discrete Mathematics, Math Conjectures, Math Proofs, Mathematical Analysis, Mathematical Modeling, Number Theory, Poetry, Probabilistic Modeling, The Art of Problem Solving





Algorithm for the Collatz Conjecture.

[***"In short the Collatz problem is simple enough that anyone can understand it, and yet relates not just to number theory (as described in other answers) but to issues of decidability, chaos, and the foundations of mathematics and of computation. That's about as good as it gets for a problem even a small child can understand." — Matt.***]

[**Is the Collatz conjecture true?**]

"We must know. We will know!" — [**David Hilbert.**]

Remark: If n = 1, the algorithm terminates all computations.

Crucial Idea: $\mathbf{R}^+$ is a dense set.

Remark: $\mathbf{R}^+$ is the set of all positive real numbers.

We claim

1. $n_t = (\frac{3}{2})^{\frac{t}{2}} * (\frac{3}{4})^{\frac{t}{4}} * (\frac{3}{8})^{\frac{t}{8}} * \ldots * (\frac{3}{2^k})^{\frac{t}{2^k}} * r = 1$

over the Collatz sequence of positive odd integers, from $n_0 > 1$ to $n_t = 1$ where the index, t, is the number of trials it takes the Collatz sequence of odd integers to converge to one.

Remark: There are no *infinite (nontrivial) cycles* of any length ($(\frac{3}{2})^{\frac{t}{2}} * (\frac{3}{4})^{\frac{t}{4}} * (\frac{3}{8})^{\frac{t}{8}} * \ldots * (\frac{3}{2^k})^{\frac{t}{2^k}} \to 0$ and $r \to \infty$ as $t \to \infty$) in the Collatz sequence since the index, t, is clearly finite in equation one.

In addition, our claim is fundamentally based on the distribution of *maximum* divisors, $2^{i_{max}}$, for *finite sets* of *consecutive* positive even integers (e.g. {2, 4, 6, ..., e_{max}}) where 2 is the minimum positive even integer and where e_{max} is the maximum positive even integer belonging to those sets...

The positive real number, r, is determined by *the algorithm* for the Collatz conjecture. And therefore, its computation is generally complicated because we cannot easily compute the maximum positive even integer, e_{max}, in the Collatz sequence (orbit) for many odd positive integers, $n_0 > 2^{68}$.

Question: For any odd positive integer, $n_0 > 2^{68}$, can we approximate the maximum positive even integer, e_{max}, in the Collatz sequence (orbit)?

The variable, k, is determined by the maximum positive even number, e_{max}, in the Collatz sequence: $k = \left\lfloor \frac{log(e_{max})}{log(2)} \right\rfloor$.

Can we refute our analysis?

We assume maximum divisors, 2^i, for any positive even integer.

Remark: The Collatz conjecture is true for all positive integers, $n_0 \leq 2^{68}$.

For any positive odd integer, $n_0 > 2^{68} - 1$, we have

$n_1 = \frac{3n_0+1}{2^{i_1}} = \frac{3}{2^{i_1}} * r_1$ where $r_1 = n_0 + \frac{1}{3}$.

$$n_2 = \frac{3(\frac{3n_0+1}{2^{i_1}})+1}{2^{i_2}} = \frac{3}{2^{i_1}} * \frac{3}{2^{i_2}} * r_2$$

where $r_2 = \frac{1}{9}(2^{i_1} + 9n_0 + 3)$.

$$n_3 = \frac{3(\frac{3(\frac{3n_0+1}{2^{i_1}})+1}{2^{i_2}})+1}{2^{i_3}} = \frac{3}{2^{i_1}} * \frac{3}{2^{i_2}} * \frac{3}{2^{i_3}} * r_3$$

where $r_3 = \frac{2^{i_1}}{27} * (2^{i_2} + 3) + n_0 + \frac{1}{3}$.

...

$n_t = r_t * 3^t \prod_{j=1}^{t}(\frac{1}{2^{i_j}})$ where $r_t > 1$ since $3^t \prod_{j=1}^{t}(\frac{1}{2^{i_j}}) < 1$ and since $n_t \geq 1$.

Question: Is $3^t \prod_{j=1}^{t}(\frac{1}{2^{i_j}}) > 1$ possible? **No!**

Why? Hint: $n_0 > 1$.

Therefore, $r_t \geq 3^{-t} \frac{1}{\prod_{j=1}^{t}(\frac{1}{2^{i_j}})}$

Thus, $r_t = 3^{-t} \frac{1}{\prod_{j=1}^{t}(\frac{1}{2^{i_j}})}$ or $r_t > 3^{-t} \frac{1}{\prod_{j=1}^{t}(\frac{1}{2^{i_j}})}$

Moreover, $n_t = r_t * 3^t \prod_{j=1}^{t}(\frac{1}{2^{i_j}}) = r * (\frac{3}{2})^{\frac{t}{2}} * (\frac{3}{4})^{\frac{t}{4}} * (\frac{3}{8})^{\frac{t}{8}} * \ldots * (\frac{3}{2^k})^{\frac{t}{2^k}} \geq 1$ where $r_t > r$.

We conclude $r_t = 3^{-t}\frac{1}{\prod_{j=1}^{t}(\frac{1}{2^{i}j})}$ since r_t is a positive *rational* number and since there are no infinite (nontrivial) cycles in any Collatz sequence. Hence $n_t = 1$.

Important Remark: **The Collatz Conjecture is true!** 🙂

Example: If we let $n_0 = 57$, then we compute $e_{max} = 196$, $k = 7$, and $t = 10$.

Therefore, $r = r(57, 10) = 1/.0841394 = 11.8850384$.

Dave's Conjecture: $r = r(n_0, t) =$ **O**(t) or $r = c_t * t$ for some real number, c_t, such that either $c_t > 1$ or $0 < c_t < 1$.

Example: If we have $t = 1$ for some n_0, then as $k \to \infty$,

$r = \frac{1}{\prod_{i=1}^{k}(\frac{3}{2^i})^{\frac{t}{2^i}}} \to \frac{4}{3} = c_1$.

Therefore, $r = r(n_0, 1) \approx \frac{4}{3}$ for infinitely many positive odd integers, $n_0 > 1$.

Remark: For our example, we compute $n_0 \in$ {5, 21, 85, 341, ..., $\frac{2^{2j}-1}{3}$, ...} for all positive integers, $j > 1$.

Questions: What are the values (convergent) for $c_2, c_3, c_4, \ldots$?

Example: We compute $r = r(n_0, 2) \approx 1\frac{7}{9}$ for infinitely many positive odd integers, $n_0 > 1$.

And therefore, $c_2 = \frac{8}{9}$ where

$n_0 \in$ {7281, 29125, 116501, 466005,..., $4 * l_j + 1$, ...} for all positive integers, $j \geq 4$.

Remark: For our example, we assume $l_1 = 7,281, l_2 = 29,125, l_3 = 116,501, l_4 = 466,005$.

Old Example: If we let $n_0 = 57$, then we compute $e_{max} = 196$, $k = 7$, and $t = 10$.

Therefore, $r = r(57, 10) = 1/.0841394 = 11.8850384$. However,

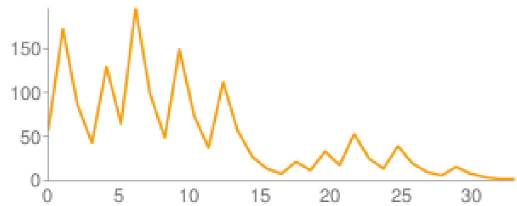
$r = r(n_0, 10) \approx \frac{1}{\prod_{i=1}^{\infty}(\frac{3}{2^i})^{\frac{10}{2^i}}} = \frac{1}{.0563135} \approx 17.75773127$

for infinitely many positive odd integers, $n_0 > 1$.

And therefore, $c_{10} \approx 1.775773127$ where

$n_0 \in$ {57, 229, 917, 3669,..., $4 * l_j + 1$, ...} for all positive integers, $j \geq 4$.

Remark: For our old example, we assume $l_1 = 57, l_2 = 229, l_3 = 917, l_4 = 3669$.



For $n_0 = 57$, we compute the following Collatz sequence where $t = 10$:

172

86

#1: 43

130

#2: 65

196

98

#3: 49

148

74

#4: 37

112

56

28

14

#5: 7

22

#6: 11

34

#7: 17

52

26

#8: **13**

40

20

10

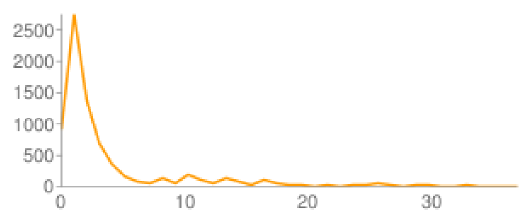
#9: **5**

16

4

2

#10: **1**



For $n_0 = 917$, we compute the following Collatz sequence where $t = 10$:

2752

1376

688

344

172

86

#1: **43**

130

#2: **65**

196

98

#3: 49

148

74

#4: 37

112

56

28

14

#5: 7

22

#6: 11

34

#7: 17

52

26

#8: 13

40

20

10

#9: 5

16

8

4

2

#10: 1

Remark: The common Collatz sequence of positive odd integers for our old example is

{43, 65, 49, 37, 7, 11, 17, 13, 5, 1}.

Final Remark (Hmm): The $3n_0 + 1$ Problem (Collatz Conjecture) has $n_0 \equiv \pm 1$ (mod 4) solutions for a given index, t. *The solutions are not unique for a given index, t.* We assume $n_0 \leftarrow 4n_0 \pm 1$.

Important Note:

All odd integers > 1 are either in the sequence, 5, 9, 13, 17, ..., 4n +1 or in the sequence, 3, 7, 11, 15, ..., 4n – 1 where n > 0.

*****The End of Our Brief Analysis of the Collatz Conjecture *****

Reference Link:

Wolfram Cloud (Mathematica Online);

The function, **nvalue[t]**, computes a random positive odd integer, $21 \leq n_0 < 100,000$ (initially), for a small index, $1 \leq t \leq 10$ (number of trials for a Collatz sequence of odd integers to converge to one from the **computed** n_0).

Source Code:

```
nvalue[t_] := (
tt = t;
icnt = 0;
n = 2 * RandomInteger[{10, 50000}] + 1;
While[icnt != tt,
While[n != 1,
If[icnt == 0, nstart = n];
n = 3n + 1;
While[EvenQ[n], n = n/2];
icnt = icnt + 1;
If[icnt > tt, n = nstart + 2 *RandomInteger[{1,10}]]
If[icnt > tt, icnt = 0]]];
Return[{tt, nstart}])
```

Some Examples:

nvalue[2] computes for **t = 2** a random value, $n_0 = 54,613$;

nvalue[10] computes for **t = 10** a random value, $n_0 = 67,077$;

nvalue[5] computes for **t = 5** a random value, $n_0 = 7,885$.

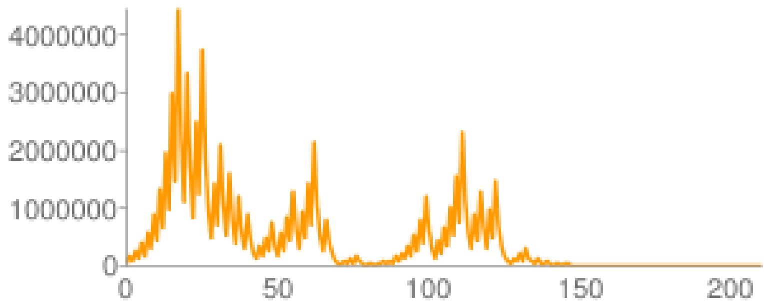
nvalue[6] computes for **t = 6** a random value, $n_0 = 82,485$.

nvalue[4] computes for **t = 9** a random value, $n_0 = 69,965$.

nvalue[1] computes for **t = 1** a random value, $n_0 = 87,381$.

nvalue[41] computes for **t = 41** a random value, $n_0 = 29,137$.

nvalue[75] computes for **t = 75** a random value, $n_0 = 57,855$.



Graph of Collatz Sequence for $n_0 = 57,855$**.**

nvalue[75] computes for **t = 75** a random value, $n_0 = 15,051$.

nvalue[150] computes for **t = 150** a random value, $n_0 = 2,139,623$.

nvalue[150] computes for **t = 150** a random value, $n_0 = 2,143,079$.

nvalue[200] computes for **t = 200** a random value, $n_0 = 6,899,327$.

nvalue[211] computes for **t = 211** a random value, $n_0 = 21,069,775$.

nvalue[250] computes for **t = 250** a random value, $n_0 = 46,912,975$.

nvalue[300] computes for **t = 300** a random value, $n_0 = 330,552,319$.

nvalue[351] computes for **t = 351** a random value, $n_0 = 1,505,110,398,721,395$.

nvalue[351] computes for **t = 351** a random value, $n_0 = 1,514,992,426,990,283$.

nvalue[408] computes for **t = 408** a random value, $n_0 = 319,762,236,301,419,029$.

...

Go Blue! 👍✌️

Relevant Reference Links:

Wolfram Alpha Computation (r_3);

Collatz Conjecture Calculator;

Our_Response_to_4.2_A_probabilistic_heuristic;

Proof of Collatz Conjecture;

The Collatz Equation that supports the Collatz Conjecture;

What is the importance of the Collatz conjecture?

An Analysis of the Collatz Conjecture;

THE 3x + 1 PROBLEM: AN OVERVIEW.

"Counting and ordering stuff (objects, sets, numbers, spaces, etc.) are fundamental."

P.S. FIGHT SEXISM AND RACISM IN THE SCIENCES INCLUDING MATHEMATICS! THANK YOU!

Oops! The Proceedings of the London Mathematical Society rejected the paper, "A Brief Analysis of the Collatz Conjecture", for publication! Why?

We are very confident our work is valid, and we suspect our work was rejected because of *political reasons*… It does happen (ostracism, blacklisting, injustice, etc.). But we are also very grateful that Lord GOD is our greatest protector, greatest provider, and our greatest redeemer. Amen!

More Links:

Two Important Properties of Convergent Collatz Sequences.

Bakuage Offers Prize of 120 Million JPY to Whoever Solves Collatz Conjecture, Math Problem Unsolved for 84 Years.

Dave.

Is the Collatz conjecture true?

We hope our work here has helped you to decide your definitive answer.

- [ ] Yes! The Collatz conjecture is true!
- [ ] The Collatz conjecture is not true!
- [ ] I do not know!

Submit

Dave September 17, 2020
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Collatz Conjecture, Discrete Mathematics, Math Conjectures, Math Proofs, Mathematical Analysis, Mathematical Modeling, Number Theory, Poetry, Probabilistic Modeling, The Art of Problem Solving

Published by Dave

"May all beings be happy and peaceful. Thank you, Lord GOD! Praise you, Lord GOD! Love you, Lord GOD! Trust you, Lord GOD! Bless you, Lord GOD! Amen!" 😂 P.S. Go Blue! Go Warriors! 👍 ✌️ View more posts

15 thoughts on “A Brief Analysis of the Collatz Conjecture”

[Sparky](#)
January 6, 2021 at 9:10 am

This is brilliant 👌
It took me several reviews to grasp what was being conveyed here. My mind is completely blown !

★ Liked by 1 person

Reply

Dave
January 6, 2021 at 10:46 am

Sparky,

Thank you! 🙂

★ Liked by 1 person

Reply

Clara Zittel
August 24, 2021 at 1:00 pm

I just added this web site to my rss reader, great stuff. Cannot get enough!

★ Liked by 1 person

Reply

Dave
September 7, 2021 at 12:11 pm

Thank you! 👍

★ Like

Reply

Brendon Scheible
August 28, 2021 at 2:56 pm

Wow that was unusual. I just wrote an very long comment but after I clicked submit my comment didn't appear. Grrrr… well I'm not writing all that over again. Anyhow, just wanted to say excellent blog!

★ Liked by 1 person

Reply

Dave
September 7, 2021 at 12:12 pm

Thank you! 👍

★ Like

Reply

Victor Kimel
August 30, 2021 at 9:32 pm

Hello I am so happy I found your weblog, I really found you by error, while I was researching on Google for something else, Anyhow I am here now and would just like to say many thanks for a marvelous post and a all round interesting blog (I also love the theme/design), I don't have time to browse it all at the moment but I have saved it and also included your RSS feeds, so when I have time I will be back to read more, Please do keep up the great job.

★ Liked by 1 person

Reply

Dave
September 7, 2021 at 12:13 pm

Thank you! 👍

★ Like

Reply

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https://j-website.net
September 3, 2021 at 11:58 am

Best view i have ever seen !

Liked by 1 person

Reply

Dave
September 7, 2021 at 12:14 pm

Thank you! 👍

Like

Reply

Loreta Sotak
December 18, 2021 at 2:13 pm

You should be a part of a contest for one of the greatest websites on the web. I will recommend this blog!

Liked by 1 person

Reply

Joesph Mcgrane
December 22, 2021 at 5:16 pm

Way cool! Some extremely valid points! I appreciate you penning this article plus the rest of the site is really good.

Liked by 1 person

Reply

Aleta Dunsford
December 25, 2021 at 10:51 am

I was able to find good advice from your articles.

Liked by 1 person

Reply

Dave

December 26, 2021 at 7:55 pm

Quote of the Day:

"In regard to the Collatz conjecture, what's true for one positive odd integer (either 4n + 1 or 4n – 1 where n > 0) is true for all positive odd integers…"

Do you agree? Why?

★ Liked by 1 person

Reply

Antoine Fullwood

February 14, 2022 at 11:48 am

This blog was… how do you say it? Relevant!! Finally I have found something which helped me. Cheers!

★ Liked by 1 person

Reply

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