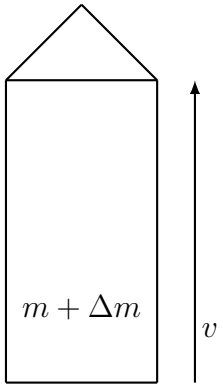
3. Rocket

In tutorials, we developed an ODE describing the performance of a rocket. We concluded that a craft's final velocity was linearly proportional to its exhaust velocity, and logarithmically proportional to the fraction of its initial mass devoted to fuel.

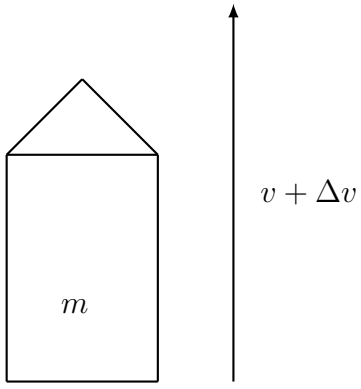
However, we based our analysis on a very simple model which neglected gravity. This might be appropriate for rocket engines in deep space, but makes no sense for space-ships taking off from Earth.

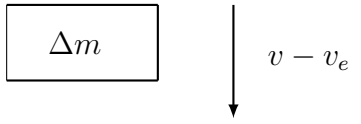
Consider now a rocket of mass $m + \Delta m$ travelling directly "upward," against a constant force of gravity g, with velocity v.



Continuing the logic introduced in tutorials, suppose the rocket fires its engines in a discrete fashion, ejecting exhaust with a mass Δm directly beneath it with velocity v_e relative to the rocket.

After this, the rocket's new velocity is $v + \Delta v$.





a) Suppose it takes Δt seconds for the rocket to eject its exhaust.

Taking gravity into account, find the change in velocity (Δv) of the rocket after this time.

b) Suppose now the rocket ejects its exhaust not in one discrete lump Δm, but as a continuous process, with a constant flow of exhaust of mass k kilograms/second.

Find $m(t)$ if $m(0) = m_0$.

c) Using a) and b), generate an ODE describing the rocket's velocity in terms of time.

d) Solve this ODE assuming an initial velocity of 0.

e) When the rocket expends all its propellant, its final mass is m_f. What is the craft's velocity at this point?

Suppose v_e, g, m_0, and m_f were all fixed by the rocket's design. What remaining parameter could you change to maximise the rocket's final velocity?

f) In this question, we've taken a simplified model of rocketry and made it more realistic by incorporating the concept of "gravity " loss.

There are a number of other practicalities to consider when launching a rocket from Earth. They all give a more complete picture of rocketry, but introduce additional complexities.

Briefly discuss two of these.