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Cooking Cosmos

Collatz Conjecture-the simplest impossible problem

Nov 9, 2020 3x+1 problem, Collatz Conjecture, Hailstone numbers

In 1937, two years after receiving his PhD degree, German mathematician Lothar Collatz (July 6, 1910 – September 26, 1990) posed a problem which remains unsolved till today. The problem can be stated as follow:

Take any positive integer, if it is even halve it, if odd multiply by three and add 1. Repeated application of the procedure will always end up in 1.

The problem is known as Collatz conjecture, a conjecture as it is yet to be proved. Numerical computation however verified the conjecture for every number less than $5{:}78\times10^{18}$. However, this does not prove anything as the very next number can falsify the conjecture. While a student at Gottingen, Collatz was interested in graphical representation of iteration of functions. In his 1930's notebooks we find iteration of arithmetic functions of similar kinds and possibly, around that time the particular problem was devised. The problem got popularized in 1950. Collatz was attending an International mathematical conference in Cambridge. He mentioned the problem to fellow participants. The problem interested several mathematicians and their names are now associated with the problem. Thus in addition to Collatz conjecture or problem, the problem is also known as the Ulam conjecture (after the Polish-American mathematician and nuclear physicist Stanisław Ulam), Kakutani's problem (after the Japanese-American mathematician Shizuo Kakutani), the Thwaites conjecture (after the English

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mathematician and educator Sir Bryan Thwaites), Hasse's algorithm (after the German mathematician Helmut Hasse), or the Syracuse problem (presumably some mathematicians from Syracuse were particularly interested in the problem and contributed to its alternate name Syracuse problem). It is now also known as the 3*n* +1 (or 3x+1) problem or conjecture.

In mathematical notation, Collatz Conjecture can be expressed as,

$$f(n) = \begin{cases} \frac{n}{2}, & if\ n\ is\ even, \\ 3n+1, & if\ n\ is\ odd. \end{cases}$$

The conjecture can be tested easily, for example, for n=10, repeated application of the function generates,

10, 5, 16, 8, 4, 2, 1.

For n=11, we get a longer sequence,

11, 34, 17, 52, 26, 13, 40, 20, 10, 5, 16, 8, 4, 2, 1.

For n=27, much longer sequence (111 terms and reaching as high as 9232 before descending to 1) is obtained,

27, 82, **41**, 124, 62, **31**, 94, **47**, 142, **71**, 214, **107**, 322, **161**, 484, 242, **121**, 364, 182, **91**, 274, **137**, 412, 206, **103**, 310, **155**, 466, **233**, 700, 350, **175**, 526, **263**, 790, **395**, 1186, **593**, 1780, 890, **445**, 1336, 668, 334, **167**, 502, **251**, 754, **377**, 1132, 566, **283**, 850, **425**, 1276,