Given the equation: $-m\,g + k\,v^2 = m\,a$

A) What is the time it takes for a golf ball dropped from an elevation of 1000 feet to hit the ground?

B) What is the velocity of the ball when it hits the ground?

C) What is the terminal velocity of the golf ball?

A) What is the time it takes for a golf ball dropped from an elevation of 1000 feet to hit the ground?

$$\sum F = -m\,g + F_d = m\,a$$

$$F_d = \frac{(CD)\,(\rho_{air})\left(\frac{\pi\,DIA^2}{4}\right)}{2}\,v^2 = k\,v^2$$

$$k = \frac{(0.25)\,(.00238)\left(\frac{\pi\,(.14^2)}{4}\right)}{2} \approx 4.6 \times 10^{-6}$$

Note that the drag coefficient Cd varies per Reynolds Number (Re), and Re is proportional to velocity (and other factors). Cd=0.25 is the approximate golf ball drag coefficient at terminal velocity with the specified k terms & equating to the Re equation. For simplicity, Cd=0.25 will be constant, which will yield a slightly lower time t to hit the ground.

$$\frac{dv}{dt} = -g + \frac{k\,v^2}{m}$$

Above equation is a 1st order differential equation. Can use non-calculus numerical method "Runge-Kutta 4th order" to solve for v(t) with step size h=0.5 s, and plot points. Then can create an approximating polynomial equation using "Lagrange Interpolating Polynomial Method". With the v(t) Lagrange polynomial, can easily integrate v(t) to get y(t) with the constant c=100 feet. This method works pretty good when the above differential equation gets nasty and can't solve directly, such as the case if we introduce Cd(v)… a varying drag coefficient equation as a function of velocity (it's not a linear equation); or if you analyze the golf ball trajectory hit at an angle say 15 deg. @180 mph and include the ball spin lift force… get two differential equations and each equation has Vx(t) and Vy(t) both squared… gets complicated fast, and numerical methods need to be used (Runge-Kutta).

Since this differential equation can be solved directly using say "Separation of Variables Method, use this method…

$$dt = \frac{1}{-g + \frac{k\,v^2}{\frac{W}{g}}}\,dv$$

$$t = \int \frac{1}{-g + \frac{k\,v^2}{\frac{W}{g}}}\,dv = -\frac{\sqrt{W}\,\mathrm{ArcTanh}\left[\frac{\sqrt{k}\,v}{\sqrt{W}}\right]}{g\,\sqrt{k}} + c \qquad @t = 0,\; v = 0 \longrightarrow c = 0$$

$$t = -\frac{\sqrt{W}\,\mathrm{ArcTanh}\left[\frac{\sqrt{k}\,v}{\sqrt{W}}\right]}{g\,\sqrt{k}}$$

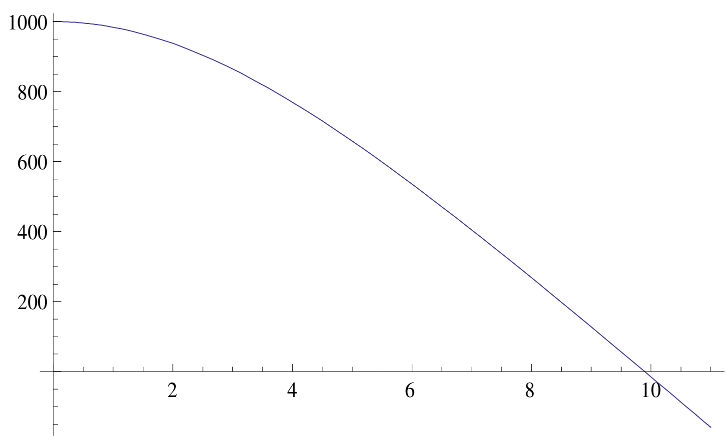
Solving for v yields:

$$v = -\frac{\sqrt{W}\ \mathrm{Tanh}\left[\frac{g\,t\,\sqrt{k}}{\sqrt{W}}\right]}{\sqrt{k}}$$

$$y = \int v\,dt = -\frac{W\,\mathrm{Log}\left[\mathrm{Cosh}\left[\frac{g\,t\,\sqrt{k}}{\sqrt{W}}\right]\right]}{g\,k} + c$$

$$@t = 0,\quad y = y_o \longrightarrow c = y_o$$

$$y = -\frac{W\,\mathrm{Log}\left[\mathrm{Cosh}\left[\frac{g\,t\,\sqrt{k}}{\sqrt{W}}\right]\right]}{g\,k} + y_o = 1000 - 675.757862\ \ \mathrm{Log}[\mathrm{Cosh}[0.218188\ \ t]]$$



y(t) graph

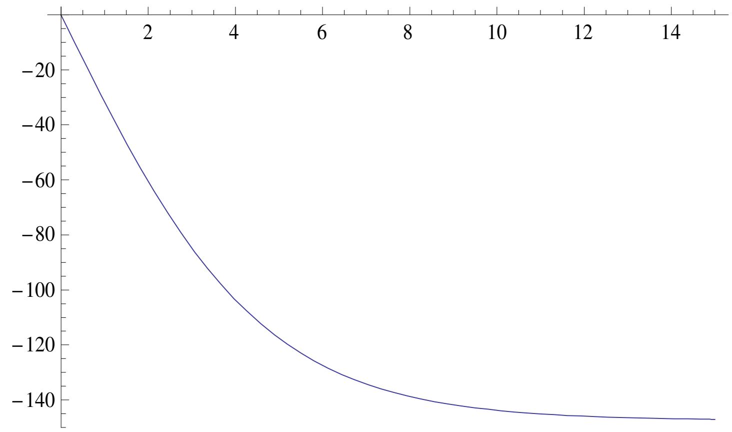
With y=0, time t is:

$$t = \frac{\sqrt{W}\ \mathrm{ArcCosh}\left[e^{\frac{g\,k\,y_o}{W}}\right]}{g\,\sqrt{k}} = 9.899\ \ s \longleftarrow \text{Answer}$$

B) What is the velocity of the ball when it hits the ground?

$$v = -\frac{\sqrt{W}\ \mathrm{Tanh}\left[\frac{g\,t\,\sqrt{k}}{\sqrt{W}}\right]}{\sqrt{k}} = -147.44196\ \ \mathrm{Tanh}[0.218188\ \ t]$$

$$v\,(9.899) = -143.57\ \ ft/s\ \ (98\ mph) \longleftarrow \text{Answer}$$



v(t) graph

C) What is the terminal velocity of the golf ball?

$$v = -\frac{\sqrt{W}\ \text{Tanh}\left[\frac{g\,t\,\sqrt{k}}{\sqrt{W}}\right]}{\sqrt{k}}$$

$$\text{as } t \longrightarrow \infty, \text{ the term : Tanh}\left[\frac{g\,t\,\sqrt{k}}{\sqrt{W}}\right] \longrightarrow 1.0$$

$$\text{Terminal Velocity} = -\sqrt{\frac{W}{k}} = -147.44 \text{ ft / s } (100.5 \text{ mph}) \longleftarrow \text{Answer}$$

A good exercise is to analyze a golf ball shot vertical upward @say v=180 mph, then get:

$$\sum F = -m\,g - F_d = m\,a$$

Can use same methods as above. When solving both directions, can find that the impact velocity will be less than the initial velocity, and the time t needed to attain maximum height is less than the time it takes to fall from this height.